IIT JEE PHYSICS
(1978–2016: 39 Years)
Topic-wise Complete Solutions

Volume II
Heat, Electromagnetism and Modern Physics

Jitender Singh
Shraddhesh Chaturvedi

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We dedicate this book to the hundreds of anonymous professors at IITs who formulated the challenging problems for IIT-JEE. The book is a showcase of their creation.

Foreword

Physics starts with observing the nature. The systematic observation results in simple rules which unlock the doors to the nature's mystery. Having learned a handful of simple rules, we can combine them logically to obtain more complicated rules and gain an insight into the way this world works. The skill, to apply the theoretical knowledge to solve any practical problem, comes with regular practice of solving problems. The aim of the present collection of problems and solutions is to develop this skill.

IIT JEE questions had been a challenge and a center of attraction for a big section of students at intermediate and college level. Independent of their occurrence as an evaluation tool, they have good potential to open up thinking threads in mind. Jitender Singh and Shraddhesh Chaturvedi have used these questions to come up with a teaching material that can benefit students. The explanations accompanying the problems could bring conceptual clarity and develop the skills to approach any unseen problem, step by step. These problems are arranged in a chapter sequence that is used in my book Concepts of Physics. Thus a student using both the books will find it as an additional asset.

Both Jitender Singh and Shraddhesh Chaturvedi have actually been my students at IIT, Kanpur. Jitender Singh has been closely associated with me since long. It gives me immense pleasure to see that my own students are furthering the cause of Physics education. I wish them every success in this work and expect much more contribution from them in future!

Dr. H C Verma
Professor of Physics
IIT Kanpur

Preface

This book provides a comprehensive collection of IIT JEE problems and their solutions. We have tried to keep our explanations simple so that any reader, with basic knowledge of intermediate physics, can understand them on his/her own without any external assistance. It can be, therefore, used for self-study.

To us, every problem in this book, is a valuable resource to unravel a deeper understanding of the underlying physical concepts. The time required to solve a problem is immaterial as far as Physics is concerned. We believe that getting the right answer is often not as important as the process followed to arrive at it. The emphasis in this text remains on the correct understanding of the principles of Physics and on their application to find the solution of the problems. If a student seriously attempts all the problems in this book, he/she will naturally develop the ability to analyze and solve complex problems in a simple and logical manner using a few, well-understood principles.

For the convenience of the students, we have arranged the problems according to the standard intermediate physics textbook. Some problems might be based on the concepts explained in multiple chapters. These questions are placed in a later chapter so that the student can try to solve them by using the concept(s) from multiple chapters. This book can, thus, easily complement your favorite text book as an advanced problem book.

The IIT JEE problems fall into one of the nine categories: (i) MCQ with single correct answer (ii) MCQ with one or more correct answers (iii) Paragraph based (iv) Assertion Reasoning based (v) Matrix matching (vi) True False type (vii) Fill in the blanks (viii) Integer Type, and (ix) Subjective. Each chapter has sections according to these categories. In each section, the questions are arranged in the descending order of year of appearance in IIT JEE.

The solutions are given at the end of each chapter. If you can't solve a problem, you can always look at the solution. However, trying it first will help you identify the critical points in the problems, which in turn, will accelerate the learning process. Furthermore, it is advised that even if you think that you know the answer to a problem, you should turn to its solution and check it out, just to make sure you get all the critical points.

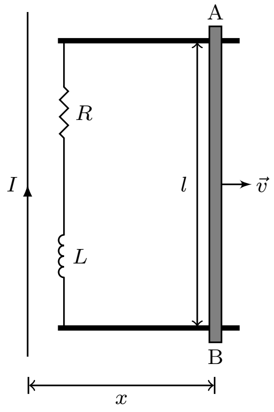
This book has a companion website, `www.concepts-of-physics.com`. The site will host latest version of the errata list and other useful material. We would be glad to hear from you for any suggestions on the improvement of the book. We have tried our best to keep the errors to a minimum. However, they might still remain! So, if you find any conceptual errors or typographical errors, howsoever small and insignificant, please inform us so that it can be corrected in the later editions. We believe, only a collaborative effort from the students and the authors can make this book absolutely error-free, so please contribute.

Many friends and colleagues have contributed greatly to the quality of this book. First and foremost, we thank Dr. H. C. Verma, who was the inspiring force behind this project. Our close friends and classmates from IIT Kanpur, Deepak Sharma, Chandrashekhar Kumar and Akash Anand stood beside us throughout this work. This work would not have been possible without the constant support of our wives Reena and Nandini and children Akshaj, Viraj and Maitreyi.

Jitender Singh, jsinghdrdo@gmail.com
Shraddhesh Chaturvedi, shraddhesh8@gmail.com

Part V

Electromagnetism

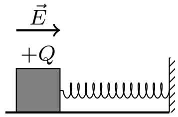


Electric Field and Potential

One Option Correct

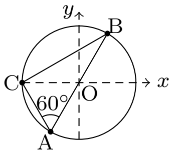
1. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly (2012)
(A) 10^{-5} V (B) 10^{-7} V (C) 10^{-9} V (D) 10^{-10} V

2. A wooden block performs SHM on a frictionless surface with frequency ν_0. The block contains a charge $+Q$ on its surface. If now, a uniform electric field $\vec{E}$ is switched on as shown, then SHM of the block will be (2011)



(A) of the same frequency and with shifted mean position.
(B) of the same frequency and with same mean position.
(C) of changed frequency and with shifted mean position.
(D) of changed frequency with same mean position.

3. Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle CAB = 60°. (2008)



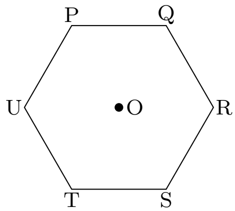
(A) The electric field at point O is $\frac{q}{8\pi\epsilon_0 R^2}$ directed along the negative x-axis.
(B) The potential energy of the system is zero.
(C) The magnitude of the force between the charge C and B is $\frac{q^2}{54\pi\epsilon_0 R^2}$.
(D) The potential at point O is $\frac{q}{12\pi\epsilon_0 R}$.

4. Positive and negative point charges of equal magnitude are kept at $\left(0,0,\frac{a}{2}\right)$ and $\left(0,0,-\frac{a}{2}\right)$, respectively. The work done by the electric field when another positive point charge is moved from $(-a,0,0)$ to $(0,a,0)$ is (2007)

(A) positive
(B) negative

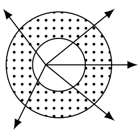
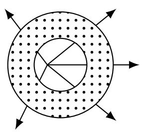
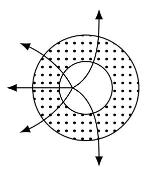
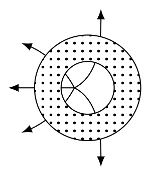
(C) zero
(D) depends on the path connecting the initial and final positions.

5. Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charge is possible for P, Q, R, S, T and U respectively, (2004)



(A) $+, -, +, -, -, +$ (B) $+, -, +, -, +, -$
(C) $+, +, -, +, -, -$ (D) $-, +, +, -, +, -$

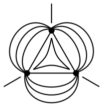
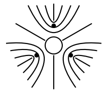
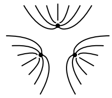
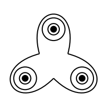
6. A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of force? (2003)

(A)  (B)  (C)  (D) 

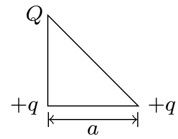
7. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to (2002)
(A) x (B) x^2 (C) x^3 (D) $1/x$

8. A uniform electric field pointing in positive x direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm, and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy (2001)
(A) $V_A < V_B$ (B) $V_A > V_B$ (C) $V_A < V_C$ (D) $V_A > V_C$

9. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in (2001)

(A)  (B)  (C)  (D) 

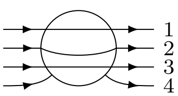
10. The dimensions of $\frac{1}{2}\epsilon_0 E^2$ (where ϵ_0 is permittivity of free space and E is electric field) is (2000)
(A) $[\mathrm{MLT^{-1}}]$ (B) $[\mathrm{ML^2T^{-2}}]$ (C) $[\mathrm{ML^{-1}T^{-2}}]$ (D) $[\mathrm{ML^2T^{-1}}]$



11. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angled isosceles triangle as shown in the figure. The net electrostatic energy of the configuration is zero, if Q is equal to (2000)
(A) $\frac{-q}{1+\sqrt{2}}$ (B) $\frac{-2q}{2+\sqrt{2}}$ (C) $-2q$ (D) $+q$

12. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$, $\cdots, \infty$ on the x-axis and a charge $-q$ is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$, $\cdots, \infty$. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\epsilon_0 r)$. Then, the potential at the origin due to the above system of charges is (1998)
(A) zero (B) $\frac{q}{8\pi\epsilon_0 x_0 \ln 2}$ (C) infinite (D) $\frac{q \ln 2}{4\pi\epsilon_0 x_0}$

13. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to (1997)
(A) 1 (B) $(m_p/m_e)^{1/2}$ (C) $(m_e/m_p)^{1/2}$ (D) 1836



14. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in the figure as (1997)

(A) 1 (B) 2 (C) 3 (D) 4

15. Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$ respectively of a x-y coordinate system. Then, (1995)
(A) the electric field E at all points on the x-axis has the same direction.
(B) work has to be done in bringing a test charge from ∞ to the origin.
(C) electric field at all point on y-axis is along x-axis.
(D) the dipole moment is $2qd$ along the x-axis.

16. Two identical thin rings, each of radius R, are coaxially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is (1992)
(A) zero (B) $\frac{(\sqrt{2}-1)q(Q_1-Q_2)}{\sqrt{2}(4\pi\epsilon_0 R)}$ (C) $\frac{\sqrt{2}q(Q_1+Q_2)}{4\pi\epsilon_0 R}$ (D) $\frac{(\sqrt{2}+1)q(Q_1+Q_2)}{\sqrt{2}(4\pi\epsilon_0 R)}$

17. A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to (1987)
(A) $-Q/2$ (B) $-Q/4$ (C) $+Q/4$ (D) $+Q/2$

18. Two equal negative charges $-q$ are fixed at points $(0, -a)$ and $(0, a)$ on y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will (1984)

(A) execute SHM about the origin.
(B) move to the origin and remain at rest.
(C) move to infinity.
(D) execute oscillatory but not SHM.

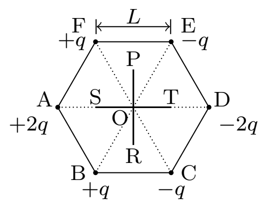
19. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is (1983)
(A) zero.
(B) 10 V.
(C) same as at a point 5 cm away from the surface.
(D) same as at a point 25 cm away from the surface.

20. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of (1981)
(A) 1 Å (B) 10^{-10} cm (C) 10^{-12} cm (D) 10^{-15} cm

One or More Option(s) Correct

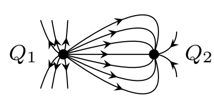
21. A length-scale (l) depends on the permittivity (ϵ) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct? (2016)
(A) $l = \sqrt{\frac{nq^2}{\epsilon k_B T}}$ (B) $l = \sqrt{\frac{\epsilon k_B T}{nq^2}}$ (C) $l = \sqrt{\frac{q^2}{\epsilon n^{2/3} k_B T}}$ (D) $l = \sqrt{\frac{q^2}{\epsilon n^{1/3} k_B T}}$

22. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\epsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct? (2012)



(A) The electric field at O is $6K$ along OD.
(B) The potential at O is zero.
(C) The potential at all points on the line PR is same.
(D) The potential at all points on the line ST is same.

23. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that (2010)



(A) $|Q_1| > |Q_2|$.
(B) $|Q_1| < |Q_2|$.
(C) at a finite distance to the left of Q_1 the electric field is zero.
(D) at a finite distance to the right of Q_2 the electric field is zero.

24. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s). (2009)

(A) The angular momentum of the charge $-q$ is constant.
(B) The linear momentum of the charge $-q$ is constant.
(C) The angular velocity of the charge $-q$ is constant.
(D) The linear speed of the charge $-q$ is constant.

25. A positively charged thin metal ring of radius R is fixed in the x-y plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is (1998)

(A) periodic for all values of z_0 satisfying $0 < z_0 < \infty$.
(B) simple harmonic for all values of z_0 satisfying $0 < z_0 \leq R$.
(C) approximately simple harmonic provided $z_0 \ll R$.
(D) such that P crosses O and continues to move along the negative z axis towards $z = -\infty$.

Paragraph Type

Paragraph for Questions 26-27

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field become zero, the electrons begin to oscillate about positive ions with natural frequency ω_p, which is called plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonate together and all the energy is reflected. This is the explanation for high reflectivity of metals. (2011)

26. Taking the electronic charge as e and permittivity as ϵ_0, use dimensional analysis to determine correct expression for ω_p.

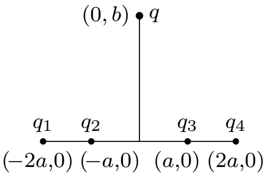
(A) $\sqrt{\frac{Ne}{m\epsilon_0}}$ (B) $\sqrt{\frac{m\epsilon_0}{Ne}}$ (C) $\sqrt{\frac{Ne^2}{m\epsilon_0}}$ (D) $\sqrt{\frac{m\epsilon_0}{Ne^2}}$

27. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electron $N = 4 \times 10^{27}\ \text{m}^{-3}$. Take $\epsilon_0 = 10^{-11}$ and $m = 10^{-30}$, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

Matrix or Matching Type

28. Four charges q_1, q_2, q_3 and q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$, and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the sign of these charges are given in *Column I*. The direction of the forces on the charge q is given in *Column II*. Match *Column I* with *Column II*.



(2014)

Column I	Column II
(P) q_1, q_2, q_3, q_4 all positive	(1) $+x$
(Q) q_1, q_2 positive; q_3, q_4 negative	(2) $-x$
(R) q_1, q_4 positive; q_2, q_3 negative	(3) $+y$
(S) q_1, q_3 positive; q_2, q_4 negative	(4) $-y$

29. Some physical quantities are given in *Column I* and some possible SI units in which these quantities may be expressed are given in *Column II*. Match the physical quantities in *Column I* with the units in *Column II*.

(2007)

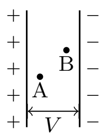
Column I	Column II
(A) GM_eM_s, where G is universal gravitational constant, M_e mass of the earth and M_s mass of the Sun.	(p) volt coulomb metre
(B) $\frac{3RT}{M}$, where R is universal gas constant, T absolute temperature and M molar mass.	(q) kg m^3 s^{-2}
(C) $\frac{F^2}{q^2B^2}$, where F is force, q charge and B magnetic field.	(r) m^2 s^{-2}
(D) $\frac{GM_e}{R_e}$, where G is universal gravitational constant, M_e mass of the earth and R_e radius of the earth.	(s) farad volt^2 kg^{-1}

True False Type

30. An electric line of force in the x-y plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1$, $y = 0$ in the x-y plane, will move along the circular line of force. (1988)

31. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a SHM along the axis of the ring. (1988)

32. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in the figure. The forces on the two protons are identical. (1986)



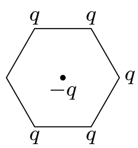
33. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulomb and the other an equal negative charge. Their masses after charging are different. (1983)

34. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If high energy X-ray beam falls on the ball, the ball will be deflected in the direction of the field. (1983)

35. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. (1981)

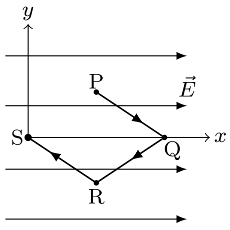
Fill in the Blank Type

36. Five point charges, each of value $+q$ coulomb, are placed on five vertices of a regular hexagon of side L metre. The magnitude of the force on the point charge of value $-q$ coulomb placed at the centre of the hexagon is ...... newton. (1992)



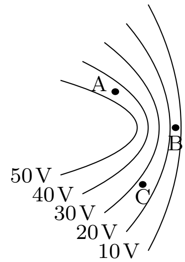
37. The electric potential V at any point x, y, z (all in metre) in space is given by $V = 4x^2$ volt. The electric field at the point $(1\text{ m}, 0\text{ m}, 2\text{ m})$ is ...... V/m. (1992)

38. A point charge q moves from point P to point S along the path $PQRS$ (see figure) in a uniform electric field E pointing parallel to the positive direction of the x-axis. The coordinates of points P, Q, R and S are $(a, b, 0)$, $(2a, 0, 0)$, $(a, -b, 0)$, $(0, 0, 0)$ respectively. The work done by the field in the above process is given by the expression ...... (1989)



39. Two small balls having equal positive charges Q (coulomb) on each are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole set-up is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the strings is ...... and the tension in each string is ...... newton. (1986)

40. Figure shows lines of constant potential in a region in which an electric field is present. The values of the potential of each line is also shown. Of the points A, B and C, the magnitude of the electric field is the greatest at the point ...... (1984)



Integer Type

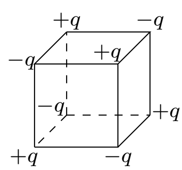
41. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of soap film is γ. The system of charges and planar film are in equilibrium and $a = k\left[q^2/\gamma\right]^{1/N}$ where k is a constant. Then N is .... (2011)

Descriptive

42. A conducting bubble of radius a and thickness t ($t \ll a$) has potential V. Now the bubble collapses into a droplet. Find the potential of the droplet. (2005)

43. A positive point charge q is fixed at origin. A dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along positive x-axis. Find (a) the kinetic energy of the dipole when it reaches a distance r from the origin and (b) force experienced by the charge q at this moment. (2003)

44. Eight point charges are placed at the corners of a cube of edge a as shown in the figure. Find the work done in disassembling this system of charges. (2003)



45. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball, so that it can make complete revolution. (2001)

46. Four point charges +8 μC, −1 μC, −1 μC, and +8 μC are fixed at the points $-\sqrt{27/2}$ m, $-\sqrt{3/2}$ m, $+\sqrt{3/2}$ m and $+\sqrt{27/2}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge +0.1 μC moves along the x direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin. Also find the kinetic energy of the particle at the origin. Assume that space is gravity free. $\left[\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \mathrm{Nm^2/C^2}.\right]$ (2000)

47. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc from a height H with zero initial velocity. The particle has $q/m = 4\epsilon_0 g/\sigma$. (1999)

(a) Find the value of H if the particle just reaches the disc.
(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

48. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $P(R\sqrt{3}, 0, 0)$ on the positive x-axis directly towards O, with an initial speed v. Find the smallest (non-zero) value of the speed v such that the particle does not return to P. (1993)

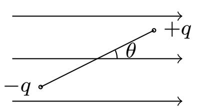
49. Answer the following questions,

(a) A charge Q is uniformly distributed over a spherical volume of radius R. Obtain an expression for the energy of the system.
(b) What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull among its constituent particles? [Assume the earth to be sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg m.]
(c) If the same charge Q as in part (a) is given to a spherical conductor of the same radius R, what will be the energy of the system? (1992)

50. Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a, 0)$ and $(+3a, 0)$ respectively in the x-y plane. (1991)

(a) Show that all points in the x-y plane where the electric potential due to the two charges is zero, lie on a circle. Find its radius and the location of its centre.
(b) Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.
(c) If a particle of charge $+q$ starts from rest at the centre of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so.

51. A point particle of mass M attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod.

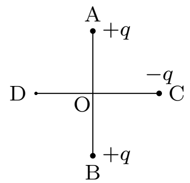


The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say about 5°) with the field direction as shown in the figure. Find the expression for the minimum time needed for the rod to become parallel to the field after it is set free. (1989)

52. Three particles, each of mass 1 g and carrying a charge q, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. [Take $g = 10\ \text{m/s}^2$.] (1988)

53. Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges? (1987)

54. Two fixed, equal, positive charges, each of magnitude $q = 5 \times 10^{-5}$ C are located at points A and B separated by a distance 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 J. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C. (1985)

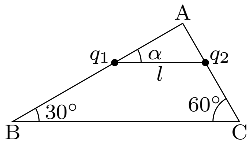


55. A thin fixed ring of radius 1 m has a positive charge 1×10^{-5} C uniformly distributed over it. A particle of mass 0.9 g and having a negative charge of 1×10^{-6} C is placed on the axis at a distance of 1 cm from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate time period of oscillations. (1982)

56. A charged particle is free to move in an electric field. It will always move along an electric line of force. (1979)

57. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} C is at rest in a horizontal uniform electric field of 20000 V/m. Find the tension in the thread of the pendulum and the angle it makes with the vertical. [Take $g = 9.8\ \text{m/s}^2$.] (1979)

58. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane as shown in the figure. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires. Considering the case when the beads are stationary determine, (1978)



(a) (i) The angle α.
(ii) The tension in the cord.
(iii) The normal reaction on the beads.

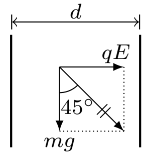
(b) If the cord is now cut what are the values of the charges for which the beads continue to remain stationary?

Answers

1. C
2. A
3. C
4. C
5. D
6. D
7. B
8. B
9. C
10. C
11. B
12. D
13. B
14. D
15. C
16. B
17. B
18. D
19. B
20. C
21. B, D
22. A, B, C
23. A, D
24. A
25. A, C
26. C
27. B
28. P↦3, Q↦1, R↦4, S↦2
29. A↦(p,q), B↦(r,s), C↦(r,s), D↦(r,s)
30. F
31. F
32. T
33. T
34. T
35. F
36. $9 \times 10^9 \frac{q^2}{L^2}$
37. $-8\hat{\imath}$
38. $-qEa$
39. $180°$, $\frac{Q^2}{16\pi\epsilon_0 L^2}$
40. B
41. 3
42. $V\left(\frac{a}{3t}\right)^{1/3}$
43. (a) $\frac{qp}{4\pi\epsilon_0 r^2}$ (b) $\frac{pq}{2\pi\epsilon_0 r^3}\hat{\imath}$
44. $\frac{5.824}{4\pi\epsilon_0}\frac{q^2}{a}$
45. 5.86 m/s
46. 3 m/s, 3×10^{-4} J
47. (a) $4a/3$ (b) $a/\sqrt{3}$
48. $\sqrt{q\lambda/(2\epsilon_0 m)}$
49. (a) $\frac{3}{20}\frac{Q^2}{\pi\epsilon_0 R}$
(b) $\frac{3}{5}\frac{GM^2}{R}$, 1.5×10^{32} J
(c) $\frac{Q^2}{8\pi\epsilon_0 R}$
50. (a) $4a$, $(5a, 0)$
(b) $V_x = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{|3a-x|} - \frac{2}{|3a+x|}\right)$
(c) $\sqrt{\frac{Qq}{8\pi\epsilon_0 ma}}$
51. $\frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$
52. 3.17×10^{-9} C
53. (a) $2q$ and $8q$ at ends, q at 3 cm from $2q$ (b) zero
54. From O 8.48 m
55. 0.628 s
56. F
57. 8.8×10^{-4} N, $27°$
58. (a) (i) $60°$ (ii) $\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{l^2} + mg$ (iii) $\sqrt{3}mg$, mg (b) $q_1 q_2 = -4\pi\epsilon_0 mgl^2$

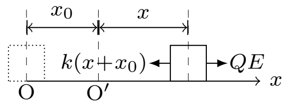
Solutions

1. The electric field in the region between the two plates is given by $E = X/d$. The proton moves at 45° to the vertical if the acceleration (resultant force) is in this direction. The resultant of electric force qE and gravitational force mg makes an angle of 45° with the vertical (see figure) if $qE = mg$ i.e., $q(X/d) = mg$. Thus,



$$X = \frac{mgd}{q} = \frac{(1.67\times10^{-27})(9.8)(1\times10^{-2})}{1.6\times10^{-19}} \approx 10^{-9}\ \text{V}.$$

2. Let m be the mass of the block, k be the spring constant, $\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (frequency of SHM when $\vec{E}$ is switched off), and O be the mean position. At the new mean position O′, the block is in equilibrium due to electrostatic and spring forces i.e.,



$$QE = kx_0, \tag{1}$$

which gives $x_0 = QE/k$. At O′, the spring is compressed by a distance x_0. Let the spring be further compressed by a distance x (see figure). Apply Newton's second law at this position to get

$$m\,\mathrm{d}^2x/\mathrm{d}t^2 = -k(x + x_0) + QE = -kx, \tag{2}$$

where we have used QE from equation (1). The equation (2) represents a SHM with frequency ν_0. Note that the net force on the block is zero at the mean position. Readers are encouraged to draw analogy of this problem with a vertically hanging spring mass system.

3. The charges at A, B, and C are $q_A = q/3$, $q_B = q/3$, and $q_C = -2q/3$. The electric fields at O due to q_A and q_B are equal in magnitude but opposite in direction. Thus, the resultant electric field at O is only due to charge q_C and is given by

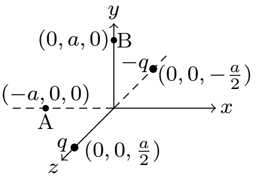
$$\vec{E}_O = -\frac{q}{6\pi\epsilon_0 R^2}\,\hat{\imath}.$$

The triangle ABC is right-angled with $\angle A = 60°$, $\angle C = 90°$, and $r_{\text{AB}} = 2R$. Thus, $r_{\text{AC}} = R$ and $r_{\text{BC}} = \sqrt{3}R$. The potential energy for the given charge distribution is

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_Aq_B}{r_{\text{AB}}} + \frac{q_Aq_C}{r_{\text{AC}}} + \frac{q_Bq_C}{r_{\text{BC}}}\right]$$
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q^2}{18R} - \frac{2q^2}{9R} - \frac{2q^2}{9\sqrt{3}R}\right] \neq 0.$$

The magnitude of force between q_C and q_B is $F_{\text{BC}} = \frac{1}{4\pi\epsilon_0}\frac{q_Bq_C}{r^2_{\text{BC}}} = \frac{q^2}{54\pi\epsilon_0 R^2}$. The potential at O is $V = \frac{1}{4\pi\epsilon_0}(q_A/R + q_B/R + q_C/R) = 0$.

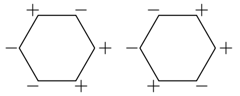
4. The charge configuration is shown in the figure. The point $A(-a,0,0)$ is at a distance $r_A = \sqrt{5}a/2$ from both the charges. Also, the point $B(-a,0,0)$ is at a distance $r_B = \sqrt{5}a/2$ from both the charges. The potentials at the point A and B are given by



$$V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r_A} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_A} = 0,$$
$$V_B = \frac{1}{4\pi\epsilon_0}\frac{q}{r_B} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_B} = 0.$$

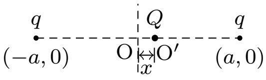
Since $V_A = V_B$, the work done in taking a unit charge from A to B is zero. The electrostatic forces are conservative and work done by them do not depend on the path. The readers are encouraged to show that work done in taking a unit charge from A to B is zero even if both the charges are positive.

5. The given condition is met if the charge at U is negative, charge at R is positive and field at O due to P, Q, S and T is zero. This is possible if the line joining the two charges and passing through O has charges of same sign on its two ends. Two such possibilities are shown in the figure.



6. The electric field inside the conductor is zero. The field lines are normal to the equipotential surface of the conductor.

7. Let O be the origin and O' be a point to the right of O at a distance x (see figure). The potentials at O and O' due to charges at $(-a,0)$ and $(a,0)$ are

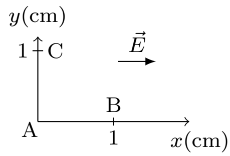


$$V_O = \frac{q}{4\pi\epsilon_0 a} + \frac{q}{4\pi\epsilon_0 a} = \frac{q}{2\pi\epsilon_0 a},$$
$$V_{O'} = \frac{q}{4\pi\epsilon_0(a+x)} + \frac{q}{4\pi\epsilon_0(a-x)} = \frac{q}{2\pi\epsilon_0}\left(\frac{a}{a^2-x^2}\right).$$

The potential energy of charge Q placed in a potential V is QV. Thus, the change in potential energy of charge Q when it is displaced by a small distance x is

$$\Delta U = QV_{O'} - QV_O = \frac{qQ}{2\pi\epsilon_0}\left[\frac{a}{a^2-x^2} - \frac{1}{a}\right]$$
$$= \frac{qQ}{2\pi\epsilon_0}\frac{x^2}{a(a^2-x^2)} \approx \frac{qQ}{2\pi\epsilon_0}\frac{x^2}{a^3}. \quad (\text{for } x \ll a).$$

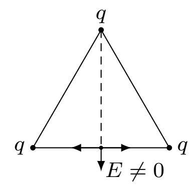
8. The uniform electric field in the region is $\vec{E} = E\,\hat{\imath}$. Let $\mathrm{d}\vec{r}_x = \mathrm{d}x\,\hat{\imath}$ and $\mathrm{d}\vec{r}_y = \mathrm{d}y\,\hat{\jmath}$ be the small displacement vectors along x and y-axes. The potentials at the point B and C relative to the point A are given by



$$V_B = V_A - \int \vec{E}\cdot\mathrm{d}\vec{r}_x = V_A - \int_0^1 E\mathrm{d}x = V_A - Ex,$$
$$V_C = V_A - \int \vec{E}\cdot\mathrm{d}\vec{r}_y = V_A. \quad (\text{since } \vec{E} \perp d\vec{r}_y).$$

Note that the potential decreases along $\vec{E}$ but does not change in a direction perpendicular to $\vec{E}$.

9. The electric field lines emanate from a positive charge. They do not intersect and do not form closed loops in electrostatics. By symmetry, the field is zero at the centroid. The fields at the middle point of each side are non-zero. The direction of electric field along the perpendicular bisector is as shown in the figure.



10. The energy density (energy per unit volume) in a region, with electric field E, is given by $\frac{1}{2}\epsilon_0 E^2$. Thus, the dimensions of $\frac{1}{2}\epsilon_0 E^2$ are same as the dimensions of the energy density which are $[\mathrm{ML^2T^{-2}}]/[\mathrm{L^3}] = [\mathrm{ML^{-1}T^{-2}}]$.

11. The electrostatic energy of charges q_1 and q_2, separated by a distance r, is given by $U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$. Electrostatic energy of the given configuration is

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{Qq}{a} + \frac{qq}{a} + \frac{Qq}{\sqrt{2}a}\right] = 0. \qquad (1)$$

Solve equation (1) to get $Q = \frac{-2q}{2+\sqrt{2}}$.

12. The potential is $V = \frac{q}{4\pi\epsilon_0 x_0}\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots\right] = \frac{q}{4\pi\epsilon_0 x_0}\ln 2$.

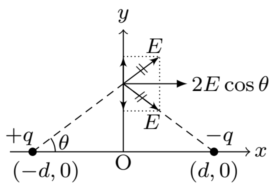
13. The magnitude of electric force on the electron and the proton is equal i.e., $F_e = F_p = qE$. The acceleration and distance travelled by the electron and proton are

$$a_e = \frac{qE}{m_e}, \quad a_p = \frac{qE}{m_p}; \quad x_e = \frac{1}{2}a_e t_1^2 = \frac{qEt_1^2}{2m_e}, \quad x_p = \frac{1}{2}a_p t_2^2 = \frac{qEt_2^2}{2m_p}.$$

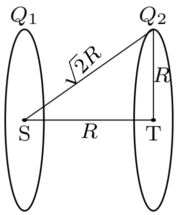
Equate $x_e = x_p$ to get $t_2/t_1 = (m_p/m_e)^{1/2}$.

14. In electrostatics (i.e., when charges are not moving or the charge density does not vary with the time), electric field inside a conductor is zero. The field lines are normal to the surface and never enter inside a conductor.

15. The electric field $\vec{E}$ on x axis is along $-\hat{\imath}$ for $x < -d$, along $+\hat{\imath}$ for $-d < x < d$, and along $-\hat{\imath}$ for $x > d$. The potential at the origin O is zero and hence no work is done in bringing a test charge from ∞ to O. The electric field at any point on the y axis is along $\hat{\imath}$ as shown in the figure. The dipole moment of the configuration is $\vec{p} = -2qd\,\hat{\imath}$.



16. Let S and T be the centres of two rings carrying charges Q_1 and Q_2, respectively (see figure). The distance of the centre from any point on the other ring is $\sqrt{2}R$. The potentials at the points S and T due to the two rings are

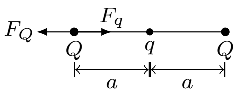


$$V_S = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}\right],$$
$$V_T = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R}\right].$$

Thus, the work done in taking a charge q from T to S is

$$W = q(V_S - V_T) = \frac{q(Q_1 - Q_2)(\sqrt{2} - 1)}{4\sqrt{2}\pi\epsilon_0 R}.$$

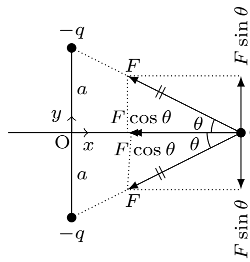
17. Let the separation between the two particles of charges Q be $2a$. Coulomb's forces on the charge q due to the other two charges are equal and opposite. Hence, charge q is always in equilibrium irrespective of its sign and magnitude. Coulomb's force on a charge Q due to another charge Q is repulsive in nature and has magnitude $F_Q = Q^2/(16\pi\epsilon_0 a^2)$. For the charge Q to be in equilibrium, Coulomb's force on it due to charge q should be attractive and of magnitude F_Q i.e.,



$$Q^2/(16\pi\epsilon_0 a^2) = -Qq/(4\pi\epsilon_0 a^2), \tag{1}$$

which gives $q = -Q/4$.

18. Let the charge Q be located at a distance x from the origin O. The electrostatic attraction forces on charge Q due to charge $-q$ located at $(0, a)$ and charge $-q$ located at $(0, -a)$ are equal in magnitude and their directions are as shown in the figure. The magnitude of electrostatic forces is



$$F = \frac{Qq}{4\pi\epsilon_0(a^2 + x^2)}.$$

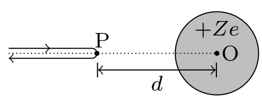
Resolve the forces along the x and y directions. The components along the y directions cancel out. The net force on charge Q is along $-x$ direction and is given by

$$F_{\text{net}} = 2F\cos\theta = \frac{2Qqx}{4\pi\epsilon_0(x^2 + a^2)^{3/2}}. \tag{1}$$

The net force on charge Q is towards the mean position O but it is not proportional to the distance from the mean position. Thus, the motion of the charge Q is not SHM. The charge Q starts moving from $(2a, 0)$ towards O, crosses the origin O and moves upto the point $(-2a, 0)$, changes the direction of motion at $(-2a, 0)$ and repeats the journey to the starting point. The users are encouraged to find the time period of oscillation.

19. The potential inside a hollow conducting sphere is constant and its value is equal to the potential at the surface. Thus, the potential at the centre is 10 V. Note that the electric field inside the hollow conducting sphere is zero.

20. Initially, kinetic energy of the alpha particle is $K_0 = 5$ MeV and its potential energy is $U_0 = 0$ (because it is far away from the nucleus). The charge of the uranium nucleus is $Q = Ze = 92e$ and charge on the alpha particle is $q = 2e$. Let O be the centre of the uranium nucleus. The alpha particle starts moving towards O and is scattered by 180° at P (see figure). The distance of the closest approach is OP $= d$. The kinetic energy of the alpha particle at P is $K_P = 0$ (since its velocity is zero) and its potential energy is



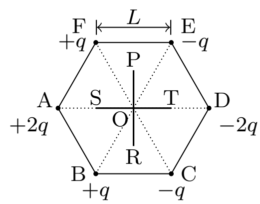
$$U_P = 2Ze^2/(4\pi\epsilon_0 d).$$

Apply conservation of energy, $K_0 + U_0 = K_P + U_P$, to get

$$d = \frac{2Ze^2}{4\pi\epsilon_0 K_0} = \frac{(9 \times 10^9)\,(2)\,(92)\,(1.6 \times 10^{-19})^2}{(5 \times 10^6)\,(1.6 \times 10^{-19})} = 5.3 \times 10^{-14}\ \text{m}.$$

21. The dimensions of thermal energy k_BT is ML^2T^{-2}. From Coulomb's law, $F = q_1q_2/(4\pi\epsilon r^2)$, the dimensions of q^2/ϵ is ML^3T^{-2}. The dimensions of number per unit volume n is L^{-3}. Substitute these dimensions in given expressions to get dimensions of $\sqrt{\frac{\epsilon k_BT}{nq^2}}$ and $\sqrt{\frac{q^2}{\epsilon n^{1/3}k_BT}}$ as L.

22. The electric field at O due to the charges at A and D is $4K$ along OD, due to the charges at B and E is $2K$ along OE and due to the charges at C and F is $2K$ along OC. For the given geometry, resultant of these fields is $6K$ along OD. The potential at O is



$$V_O = \sum \frac{1}{4\pi\epsilon_0}\frac{q_i}{L} = \frac{1}{4\pi\epsilon_0 L}\sum q_i = 0.$$

For any point on PR, we have pairs of equal and opposite charges at the same distance making the potential at any point on PR zero. It may be seen that potential at points on OS is positive and that on OT is negative. The readers are encouraged to show that the potential on ST (at a distance x from O, taken positive towards the right) is

$$V(x) = \frac{q}{4\pi\epsilon_0}\left[\frac{2}{\sqrt{L^2+x^2+xL}} - \frac{2}{\sqrt{L^2+x^2-xL}} - \frac{4x}{L^2-x^2}\right].$$

23. From the direction of electric field lines, Q_1 is positive and Q_2 is negative. The density of electric field lines (which is an indication of flux) is more around Q_1 in comparison to Q_2. In other words, the flux ϕ_1 through a small sphere containing Q_1 is more than the flux ϕ_2 through a similar sphere containing Q_2. From Gauss's law, flux $\phi = q_{\text{enc}}/\epsilon_0$. Thus, $\phi_1 > \phi_2$ implies $|Q_1| > |Q_2|$. The electric field at a distance x towards the right of Q_2 is given by

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{(d+x)^2} - \frac{1}{4\pi\epsilon_0}\frac{Q_2}{x^2},$$

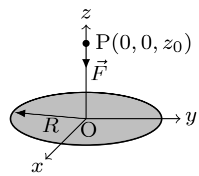
where d is the separation between Q_1 and Q_2. Since $Q_1 > Q_2$, $|\vec{E}|$ becomes zero for some finite x. The readers are encouraged to show that there are two such points given by

$$x_1 = \frac{Q_2 + \sqrt{Q_1 Q_2}}{Q_1 - Q_2}d, \qquad x_2 = \frac{Q_2 - \sqrt{Q_1 Q_2}}{Q_1 - Q_2}d.$$

Also, show that the electric field is non-zero at all finite distances towards the left of Q_1. Draw the electric field lines in the entire region.

24. The torque on a charge $-q$ due to Coulomb force $\vec{F} = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{r}$ is $\vec{\tau} = \vec{r}\times\vec{F} = \vec{0}$. Since $\vec{\tau} = \mathrm{d}\vec{L}/\mathrm{d}t = \vec{0}$, angular momentum $\vec{L}$ is a constant. In elliptical orbit, r varies and hence to keep $L = m\omega^2 r$ constant, ω must vary with r. The speed $v = \omega r = \sqrt{Lr/m}$ also varies with r. By Netwon's second law, $\vec{F} = \mathrm{d}\vec{p}/\mathrm{d}t \neq \vec{0}$ and hence $\vec{p}$ cannot be a constant. The readers are encouraged to draw analogy between this problem and the planetary motion (Kepler's laws).

25. Let Q be the charge on a ring of radius R and centre O. The electric field at a point $P(0,0,z_0)$ due to the ring is



$$\vec{E} = \frac{Qz_0}{4\pi\epsilon_0(R^2+z_0^2)^{3/2}}\,\hat{k},$$

and the force on the charge $-q$ placed at P is

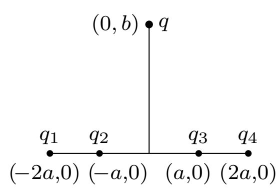
$$\vec{F} = -\frac{Qqz_0}{4\pi\epsilon_0(R^2+z_0^2)^{3/2}}\,\hat{k}.$$

This force accelerates the charge towards O (see figure). When particle crosses O and moves to the other side of the ring, force direction is again towards O. Thus, the particle executes a periodic motion about O.
If $z_0 \ll R$, the force becomes $\vec{F} \approx -\frac{Qqz_0}{4\pi\epsilon_0 R^3}\,\hat{k}$, which is proportional to the displacement z_0 and is always towards O. In this case, the particle execute SHM with frequency $\omega = \sqrt{\frac{Qq}{4\pi\epsilon_0 m R^3}}$.

26. Using Coulomb's law, $F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$, the dimensions of e^2/ϵ_0 is given by $[\mathrm{ML^3T^{-2}}]$. The number density N has dimensions $[\mathrm{L^{-3}}]$. This gives dimensions of $\sqrt{\frac{Ne^2}{m\epsilon_0}}$ as $\mathrm{T^{-1}}$ and hence $\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_0}}$.

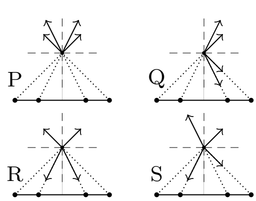
27. The plasma reflection occurs at the frequency $\omega = \omega_p$. Thus, $\lambda = 2\pi c/\omega_p = 2\pi c/\sqrt{\frac{Ne^2}{m\epsilon_0}}$, where $c = 3\times10^8$ m/s and $e = 1.6\times10^{-19}$ C. Substitute the values to get $\lambda = 589$ nm ≈ 600 nm.

28. Let $|q_1| = |q_2| = |q_3| = |q_4| = q'$. The forces on q by q_1, q_2, q_3, and q_4 in all the four cases (P, Q, R, S) are shown in the figure. Coulomb's law gives the magnitude of force by q_1 and q_4 as



$$F_{1,4} = \frac{qq'}{4\pi\epsilon_0(b^2+4a^2)},$$

and that by q_2 and q_3 as



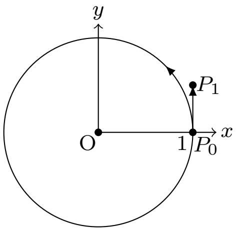
$$F_{2,3} = \frac{qq'}{4\pi\epsilon_0(b^2+a^2)}.$$

Thus, $F_{1,4} < F_{2,3}$. See the figure for the directions of forces in four cases. Resolve the forces in x and y directions and compare the magnitudes to get the answer.

29. From Newton's law of gravitation, $F = GM_eM_s/r^2$, units of GM_eM_s are same as that of Fr^2 which are kg m^3 s^{-2}. Other units of Fr^2 are energy-metre. One of the units of energy are coulomb-volt since $U = qV$. Thus, volt-coulomb-metre are also the units of Fr^2 and GM_eM_s.

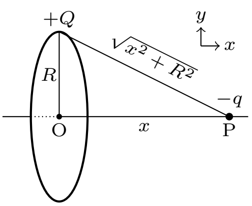
The internal energy of an ideal gas is $\frac{3}{2}nRT = \frac{3}{2}(m/M)RT$, which gives the units of $3RT/M$ as energy-kg^{-1} i.e., m^{2}s^{-2}. The energy stored in a capacitor, $U = \frac{1}{2}CV^2$, gives the units of energy as farad-volt^2. Thus, another units of $3RT/M$ are farad-volt^2 kg^{-1}.
The force on a current carrying wire in a magnetic field, $F = IlB = (q/t)lB$, gives units of $F^2/(q^2B^2)$ as m^{2}s^{-2} which are same as farad-volt^2 kg^{-1}.
The gravitational potential energy, $U = -GM_em/R_e$, gives the units of GM_e/R_e as energy-kg^{-1} which are same as m^{2}s^{-2} and farad-volt^2 kg^{-1}.

30. The electric force on a positive unit charge placed at a point P is along the tangent to electric lines of force at P. The path of the particle depends on the initial conditions (position, velocity) and acceleration. In the given case, initial position is $P_0(1,0)$, initial velocity is zero, and initial acceleration is $\vec{a} = qE/m\,\hat{\jmath}$. Thus, the particle starts moving in $\hat{\jmath}$ direction. Let the particle moves to a new position P_1 in a small time interval Δt. The position of the particle after next Δt time interval will depend on the velocity and acceleration at P_1, which cannot be deduced from the given information. Therefore, it cannot be concluded that the particle moves along the given line of force.



31. The electric field due to a uniformly charged ring of radius R and charge Q at a point P on its axis (see figure) is given by

$$\vec{E}(x) = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(R^2+x^2)^{3/2}}\,\hat{\imath}.$$



Thus, force on a negative charge $(-q)$ placed at a distance x on the axis of the ring is

$$\vec{F}(x) = -\frac{1}{4\pi\epsilon_0}\frac{Qqx}{(R^2+x^2)^{3/2}}\,\hat{\imath}. \tag{1}$$

The force is restoring in nature but it is not proportional to x. Thus, the motion of the particle is *not* SHM but periodic if x is large ($x = 2R$). The readers are encouraged to show, for $x \ll R$, that equation (1) reduces to

$$\vec{F}(x) = -\left[Q/(4\pi\epsilon_0 R^3)\right] x\,\hat{\imath}, \tag{2}$$

and the particle executes SHM with an angular frequency $\omega = \sqrt{Q/(4\pi\epsilon_0 m R^3)}$.

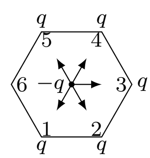
32. The magnitude of electric field between the two parallel plates of a capacitor with potential difference V is $E = V/d$, where d is the separation between the plates. Hence, forces qE on two protons (of charge q) are same irrespective of their locations within the capacitor.

33. Let M_0 be the mass of the neutral sphere, m be the mass of an electron and $-e$ be the charge on an electron. A sphere is given the positive charge Q by taking away $n = Q/e$ electrons from it. Thus, the mass of the positively charged sphere is $M_{\text{pos}} = M_0 - mn = M_0 - mQ/e$. Another sphere is given the negative charge $-Q$ by putting $n = Q/e$ electrons on it. Thus, the mass of the negatively charged sphere is $M_{\text{neg}} = M_0 + mn = M_0 + mQ/e$. The readers are encouraged to find the charges on the spheres if the difference in their masses is 1 μg.

34. The high energy X-rays cause ejection of the photoelectrons from the metal ball (photoelectric effect). Thus, the ball gets positively charged. The positively charged ball is deflected in the direction of electric field.

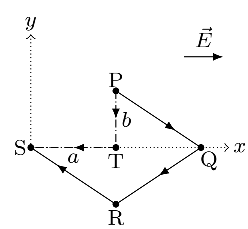
35. The electrostatic field/force is conservative in nature. The work done by a conservative force is independent of the path and depends only on the end points. Note that the work done by a conservative force in a closed path is always zero.

36. The forces on $-q$ due to charges at 1 and 4 are equal and opposite (see figure). Also, the forces on $-q$ due to charges at 2 and 5 are equal and opposite. Thus, the net force on $-q$ is due to charge at 3 and its magnitude is $F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2} = 9\times 10^9\frac{q^2}{L^2}$.



37. The electric field is given by $\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k} = -8x\,\hat{\imath}$. Substitute $x = 1$ to get $\vec{E}$ at the point $(1\text{ m}, 0\text{ m}, 2\text{ m})$ i.e., $\vec{E} = -8\,\hat{\imath}$.

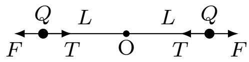
38. The work done by the conservative forces (electrostatic, gravitational, etc.) is independent of the path i.e., it depends only on the initial and final points. Thus, the work done by the field along path $P\to Q\to R\to S$ is same as the work done along the path $P\to T\to S$ (see figure). The work done by the field in moving a charge q along the path $P\to T\to S$ is given by



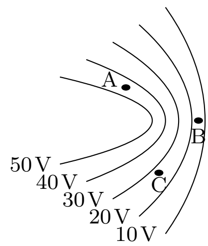
$$W = \int_P^S q\mathrm{d}V = -q\int_P^S \vec{E}\cdot\mathrm{d}\vec{l} = -q\int_P^T \vec{E}\cdot\mathrm{d}\vec{l} - q\int_T^S \vec{E}\cdot\mathrm{d}\vec{l}$$

$$= -q\int_b^0 (E\,\hat{\imath})\cdot(-\mathrm{d}y\,\hat{\jmath}) - q\int_a^0 (E\,\hat{\imath})\cdot(-\mathrm{d}x\,\hat{\imath}) = qE\int_a^0 \mathrm{d}x = -qEa.$$

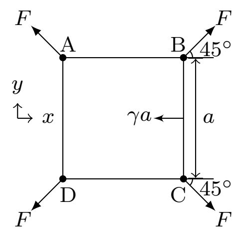
Note that the potentials at the point S and P are related by $V_P = V_S - Ea$.
Aliter: The force on the charge is $\vec{F} = qE\,\hat{\imath}$ and its displacement is $\vec{S} = \vec{r}_S - \vec{r}_P = -a\,\hat{\imath} - b\,\hat{\jmath}$. Thus, $W = \vec{F}\cdot\vec{S} = (qE\,\hat{\imath})\cdot(-a\,\hat{\imath} - b\,\hat{\jmath}) = -qEa$.



39. In the state of weightlessness, the net force on the charges are only due to Coulomb repulsion. The repulsion causes the two charges to move farthest away from each other. In this position, the angle between the two strings is 180° and separation between the two charges is $r = 2L$. The forces acting on each charge are Coulomb force $F = Q^2/(4\pi\epsilon_0 r^2) = Q^2/(16\pi\epsilon_0 L^2)$ and tension T. In equilibrium, $T = Q^2/(16\pi\epsilon_0 L^2)$.



40. The electric field is related to electric potential by $E = -\mathrm{d}V/\mathrm{d}x$. The potential difference between the successive lines of constant potential is $\Delta V = 10$ V. The perpendicular distances between successives lines at the point A and at the point C are almost equal but this distance is smaller at the point B i.e., $\Delta x_A = \Delta x_C > \Delta x_B$. Hence, $|E_B| = \Delta V/\Delta x_B > \Delta V/\Delta x_A = |E_A| = |E_C|$.



41. The force on the line BC due to surface tension is γa (see figure). The electrostatic forces on charge at B due to charges at A, D, and C are along AB, DB, and CB, respectively. Thus, the total force on the charge at B due to other three charges is

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\left[\frac{q^2}{a^2}\hat{\imath} + \frac{q^2}{a^2}\hat{\jmath} + \frac{q^2}{2a^2}\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right]$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right).$$

By symmetry, the magnitudes of force on the other three charges are same as the magnitude of force on charge at B. The directions of these forces are as shown in the figure. In equilibrium, the force due to surface tension (on line BC) is equal and opposite to x-component of electrostatic force acting on the charges placed at point B and C i.e.,

$$\gamma a = 2F\cos 45^\circ = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left[2 + 1/\sqrt{2}\right],$$

which gives

$$a = \sqrt[3]{\frac{1}{4\pi\epsilon_0}\left(2+1/\sqrt{2}\right)}\;\left[q^2/\gamma\right]^{1/3}.$$

42. The volume of liquid in a bubble of radius a and thickness t is $\mathcal{V}_b = \frac{4}{3}\pi((a+t)^3 - a^3) \approx 4\pi a^2 t$ (since $t \ll a$). The volume of liquid in the droplet of radius r is $\mathcal{V}_d = \frac{4}{3}\pi r^3$. Equate $\mathcal{V}_b = \mathcal{V}_d$ to get

$$r = (3a^2 t)^{1/3}.$$

The potential on a spherical shell of radius a and charge q is $V = q/(4\pi\epsilon_0 a)$. Thus, charge on the bubble having potential V is $q = 4\pi\epsilon_0 aV$. The charge conservation gives charge on the droplet as $q = 4\pi\epsilon_0 aV$. The potential on the droplet is given by

$$V_d = \frac{q}{4\pi\epsilon_0 r} = \frac{4\pi\epsilon_0 aV}{(3a^2t)^{1/3}} = \left(\frac{a}{3t}\right)^{1/3} V.$$

43. Total energy of a dipole $\vec{p} = p\,\hat{\imath}$ when it is far away from the charge q, is zero. Now, this dipole is placed in the electric field of charge q. The electric field of charge q and the potential energy of the dipole are given by

$$\vec{E}_q = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{\imath}, \qquad U = -\vec{p}\cdot\vec{E}_q = -\frac{qp}{4\pi\epsilon_0 r^2}.$$

The conservation of energy, $K + U = 0$, gives dipole kinetic energy as $K = -U = qp/(4\pi\epsilon_0 r^2)$. The electric field at the origin by the dipole and force on the charge q are

$$\vec{E}_p = \frac{2p}{4\pi\epsilon_0 r^3}\,\hat{\imath}, \qquad \vec{F}_q = q\vec{E}_p = \frac{2pq}{4\pi\epsilon_0 r^3}\,\hat{\imath}.$$

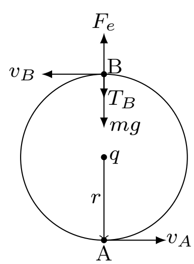
The readers are encouraged to find the force on $\vec{p}$ by differentiating its potential energy, $F_p = -\frac{\mathrm{d}U}{\mathrm{d}r}$. Apply Newton's third law to find the force on q.

44. In the given system, there are ${}^8\mathrm{C}_2 = 28$ pairs of charges. The charge pairs on cube edges (twelve edges of length a each) have unlike charges. The charge pairs on face-diagonals (six faces, two diagonals per face, twelve face-diagonal of length $\sqrt{2}a$ each) have like charges. The charge pairs on main-diagonals (four main-diagonal each with length $\sqrt{3}a$) have unlike charges. Thus, the potential energy of the system is

$$U = \frac{1}{4\pi\epsilon_0}\left[-12\frac{q^2}{a} + 12\frac{q^2}{\sqrt{2}a} - 4\frac{q^2}{\sqrt{3}a}\right] = -\frac{5.824}{4\pi\epsilon_0}\frac{q^2}{a}.$$

The work done to disassemble the system is $W = -U = \frac{5.824}{4\pi\epsilon_0}\frac{q^2}{a}$.

45. Let $q = 1\,\mu\mathrm{C}$ be the charge and $m = 2\times10^{-3}$ kg be the mass of a particle moving in a circle of radius $r = 0.8$ m. The forces acting on the particle are its weight mg downward, tension T radially inward, and electrostatic force $F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$ radially outward. Let v_A and v_B be its velocities at the lowest point A and the highest point B (see figure). At B, the radially inward force provides the centripetal acceleration i.e.,



$$mg + T_B - F_e = mv_B^2/r. \tag{1}$$

The electrostatic potential energies of the particle at A and B are same. Using the conservation of mechanical energy between A and B, we get

$$\tfrac{1}{2}mv_A^2 = \tfrac{1}{2}mv_B^2 + 2mgr. \tag{2}$$

Eliminate v_B from equations (1) and (2) to get

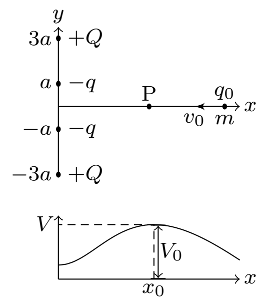
$$v_A^2 = 5rg/m + rT_B/m - rF_e/m. \tag{3}$$

Velocity v_A is minimum when $T_B = 0$ (since $T_B \geq 0$). Substitute $T_B = 0$ in equation (3) to get the minimum value of v_A as

$$v_A = \left(5rg - \frac{rF_e}{m}\right)^{1/2} = \left(5gr - \frac{1}{4\pi\epsilon_0}\frac{q^2}{mr}\right)^{1/2}. \tag{4}$$

Substitute the values of q, m, r, and g in equation (4) to get $v_A = 5.86$ m/s. The readers are encouraged to analyse the problem if $F_e > mg$.

46. Given $q = 1\ \mu\text{C} = 10^{-6}$ C, $Q = 8\ \mu\text{C} = 8\times10^{-6}$ C, $q_0 = 0.1\ \mu\text{C} = 10^{-7}$ C, $m = 6\times10^{-4}$ kg and $a = \sqrt{3/2}$ m. Consider a point P at a distance x from the origin. The potential at P due to given charge distribution is



$$V(x) = \frac{1}{4\pi\epsilon_0}\left[\frac{2Q}{\sqrt{x^2+9a^2}} - \frac{2q}{\sqrt{x^2+a^2}}\right].$$

The potential varies with x and attains its maximum at x_0 (see figure). The $V(x)$ becomes maximum when

$$\frac{\mathrm{d}V(x)}{\mathrm{d}x} = -\frac{2x}{4\pi\epsilon_0}\left[\frac{Q}{(x^2+9a^2)^{3/2}} - \frac{q}{(x^2+a^2)^{3/2}}\right] = 0. \tag{1}$$

Substitute $Q = 8q$ in equation (1) and solve to get $x_0 = \sqrt{5/3}\,a = \sqrt{5/2}$ m. The potential at x_0 is $V_0 = V(x_0) = 2.7\times10^4$ V. The kinetic energy of charge q_0 at $x = \infty$ should be sufficient to cross the potential barrier V_0,

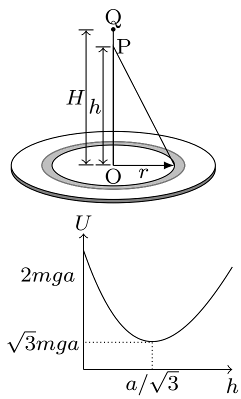
$$\tfrac{1}{2}mv_0^2 = q_0V_0, \quad\Longrightarrow\quad v_0 = \sqrt{2q_0V_0/m} = 3\ \text{m/s}.$$

The potential energy of q_0 at the origin is

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{2Qq_0}{3a} - \frac{2qq_0}{a}\right] = 2.4\times10^{-3}\ \text{J}.$$

Let K be the kinetic energy of q_0 at the origin. The conservation of energy, $\frac{1}{2}mv_0^2 = K + U$, gives

$$K = \tfrac{1}{2}mv_0^2 - U = 3\times10^{-4}\ \text{J}.$$



47. Consider a ring of radius r and width $\mathrm{d}r$. The charge on the ring is $\mathrm{d}q = 2\pi r\sigma \mathrm{d}r$, where σ is the surface charge density of the ring. The potential due to the ring at a point P located at a height h on the axis of the disc is (see figure)

$$\mathrm{d}V = \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma r \mathrm{d}r}{\sqrt{r^2+h^2}}.$$

Integrate $\mathrm{d}V$ from $r=0$ to $r=a$ to get the potential due to the complete disc

$$V = \frac{\sigma}{2\epsilon_0}\left[\sqrt{a^2+h^2}-h\right].$$

Thus, the potentials at O and Q are given by

$$V_O = \frac{\sigma a}{2\epsilon_0}, \qquad V_Q = \frac{\sigma}{2\epsilon_0}\left[\sqrt{a^2+H^2}-H\right].$$

For the particle to just reach O, decrease in its gravitational potential energy should be equal to the increase in its electrostatic potential energy i.e.,

$$mgH = q(V_O - V_Q) = \frac{q\sigma}{2\epsilon_0}\left[a + H - \sqrt{a^2+H^2}\right]. \tag{1}$$

Substitute $q/m = 4\epsilon_0 g/\sigma$ in equation (1) to get

$$\sqrt{a^2+H^2} = a + H/2, \quad \text{which gives,} \quad H = 4a/3.$$

Total potential energy (U) of the particle at point P is the sum of its gravitational and electrostatic potential energies i.e.,

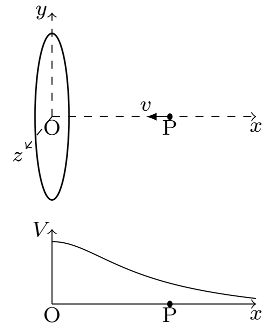
$$U = mgh + \frac{q\sigma}{2\epsilon_0}\left[\sqrt{a^2+h^2}-h\right] = mg\left[2\sqrt{a^2+h^2}-h\right].$$

The potential energy attains extremum at the equilibrium position i.e.,

$$\frac{\mathrm{d}U}{\mathrm{d}h} = mg\left[\frac{2h}{\sqrt{a^2+h^2}} - 1\right] = 0,$$

which gives $h_{\min} = a/\sqrt{3}$ and $U_{\min} = \sqrt{3}mga$. The figure shows the variation of U with height h. Note that the equilibrium is *stable* (i.e., $\mathrm{d}^2U/\mathrm{d}h^2 > 0$).

48. The charge on the ring is $Q = 2\pi R\lambda$. The potential at a point $(x, 0, 0)$ on the axis of the ring is given by



$$V(x) = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2 + x^2}}.$$

Thus, the potentials at the point $O(0, 0, 0)$ and $P(R\sqrt{3}, 0, 0)$ are

$$V_O = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}, \qquad V_P = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}.$$

The potential increases monotonically from P to O and attains its maximum value at O. The particle will not come back to P if it just crosses O. The energy required by a particle of charge q to reach potential V_O from potential V_P is $q(V_O - V_P)$. Thus,

$$\frac{1}{2}mv^2 = q(V_O - V_P) = \frac{1}{4\pi\epsilon_0}\frac{qQ}{2R} = \frac{q\lambda}{4\epsilon_0},$$

which gives $v = \sqrt{q\lambda/(2\epsilon_0 m)}$.

49. The electric field inside and outside of a uniformly charged sphere having a charge Q and radius R is given by

$$E(r) = \begin{cases} \frac{Qr}{4\pi\epsilon_0 R^3}, & \text{if } r \leq R; \\ \frac{Q}{4\pi\epsilon_0 r^2}, & \text{if } r > R. \end{cases}$$

The energy density (energy per unit volume) in space, having an electric field E, is given by $\frac{1}{2}\epsilon_0 E^2$. Let us calculate the electrostatic potential energy stored inside and outside the sphere separately.
Take a spherical shell of radius r and thickness $\mathrm{d}r$ inside the sphere. The potential energy of this shell is

$$\mathrm{d}U_1 = \frac{1}{2}\epsilon_0 E^2(4\pi r^2 \mathrm{d}r) = \frac{Q^2}{8\pi\epsilon_0 R^6} r^4 \mathrm{d}r.$$

Integrate $\mathrm{d}U_1$ from $r = 0$ to $r = R$ to get

$$U_1 = \frac{Q^2}{8\pi\epsilon_0 R^6}\int_0^R r^4 \mathrm{d}r = \frac{Q^2}{40\pi\epsilon_0 R}.$$

Similarly, the potential energy stored in a spherical shell of radius r and thickness $\mathrm{d}r$ outside the sphere is

$$\mathrm{d}U_2 = \frac{1}{2}\epsilon_0 E^2(4\pi r^2 \mathrm{d}r) = \frac{Q^2}{8\pi\epsilon_0}\frac{\mathrm{d}r}{r^2}.$$

Integrate from $r = R$ to $r = \infty$ to get the potential energy stored outside the sphere as

$$U_2 = \frac{Q^2}{8\pi\epsilon_0}\int_R^\infty \frac{\mathrm{d}r}{r^2} = \frac{Q^2}{8\pi\epsilon_0 R}.$$

Total potential energy is $U = U_1 + U_2 = \frac{3Q^2}{20\pi\epsilon_0 R}$.
In case of a spherical conductor, the electric field inside the conductor is zero giving $U_1 = 0$. The field and energy outside the conductor are

$$E = \frac{Q}{4\pi\epsilon_0 r^2}, \qquad U = U_2 = \frac{Q^2}{8\pi\epsilon_0 R}.$$

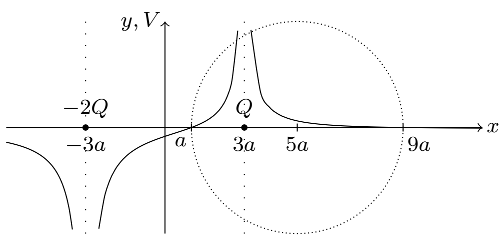
The readers are encouraged to compare this with the potential energy of a spherical capacitor of radius R and charge Q.
Many results from electrostatics can be directly used in gravitation by replacing charge Q by mass M and constant $1/(4\pi\epsilon_0)$ by universal gravitational constant G. Thus, gravitational potential energy of a sphere of mass M and radius R is $U = -\frac{3}{5}\frac{GM^2}{R}$ (negative because gravitational forces are always attractive). Hence, energy required to completely disassemble the sphere is

$$\begin{aligned} E &= -U = \frac{3}{5}\frac{GM^2}{R} = \frac{3}{5}\frac{GM}{R^2}MR = \frac{3}{5}gMR \\ &= \frac{3}{5}(9.8)(2.5\times 10^{31}) = 1.5\times 10^{32}\ \text{J}. \end{aligned}$$

50. The net electric potential at the point $P(x, y)$ due to the charge $-2Q$ located at $(-3a, 0)$ and the charge Q located at $(3a, 0)$ is given by

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{(x-3a)^2+y^2}} - \frac{2}{\sqrt{(x+3a)^2+y^2}}\right].$$

The potential is zero when $\frac{1}{\sqrt{(x-3a)^2+y^2}} = \frac{2}{\sqrt{(x+3a)^2+y^2}}$, which simplifies to $(x-5a)^2 + y^2 = (4a)^2$. This is the equation of a circle of radius $4a$ and centre $(5a, 0)$.



The potential on the x axis is given by

$$V(x) = \frac{Q}{4\pi\epsilon_0}\left(\frac{-2}{|x+3a|} + \frac{1}{|x-3a|}\right). \tag{1}$$

The modulus function is defined as $|x| = -x$ if $x < 0$ and $|x| = x$ if $x \geq 0$. Using definition of modulus function, we can write equation (1) as

$$V(x) = \begin{cases} \frac{Q}{4\pi\epsilon_0}\left(\frac{2}{3a+x} + \frac{1}{3a-x}\right), & \text{if } x \leq -3a; \\ \frac{Q}{4\pi\epsilon_0}\left(\frac{-2}{x+3a} + \frac{1}{3a-x}\right), & \text{if } -3a < x \leq 3a; \\ \frac{Q}{4\pi\epsilon_0}\left(\frac{-2}{x+3a} + \frac{1}{x-3a}\right), & \text{if } x > 3a. \end{cases} \quad (2)$$

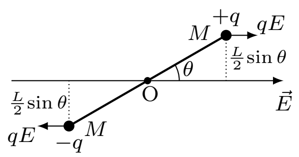
It can be seen from equation (2) that $V \to -\infty$ as $x \to -3a$ and $V \to \infty$ as $x \to 3a$. The potential is zero at $x = a$ and at $x = 9a$ (see figure). The potential at the centre of circle ($x = 5a$) is

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{-2}{8a} + \frac{1}{2a}\right) = \frac{Q}{16\pi\epsilon_0 a},$$

which has a positive value. The potential at the circumference of the circle is zero. A positive charge moves from a higher potential to a lower potential. By conservation of energy, decrease in the potential energy is equal to increase in kinetic energy i.e.,

$$\frac{1}{2}mv^2 = \frac{qQ}{16\pi\epsilon_0 a}, \quad \text{which gives} \quad v = \sqrt{\frac{Qq}{8\pi\epsilon_0 ma}}.$$

51. Coulomb's force qE acts on each charge as shown in the figure. The torque of these forces about the point O is in clockwise direction and is given by



$$\begin{aligned} \tau &= qE(L/2)\sin\theta + qE(L/2)\sin\theta \\ &= qEL\sin\theta. \end{aligned} \quad (1)$$

The torque τ is related to the angular acceleration $\alpha = -\mathrm{d}^2\theta/\mathrm{d}t^2$ (note that the torque is restoring in nature) by $I\alpha = \tau$ i.e.,

$$-I\,\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = qEL\sin\theta \approx qEL\theta, \qquad (\because \sin\theta \approx \theta \text{ for small } \theta), \quad (2)$$

where I is the moment of inertia of the rod (along with the two particles of mass M each) about an axis passing through O. As the rod is massless, the total moment of inertia is given by $I = M(L/2)^2 + M(L/2)^2 = \frac{1}{2}ML^2$. Substitute it in equation (2) to get

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{2qE}{ML}\theta = -\omega^2\theta. \quad (3)$$

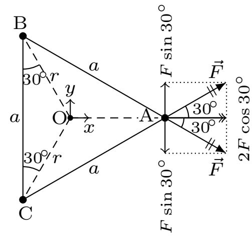
The equation (3) represents a SHM with an angular frequency $\omega = \sqrt{\frac{2qE}{ML}}$ and the time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ML}{2qE}}. \quad (4)$$

The minimum time for the rod to become parallel to the field is $t_{\min} = T/4$.

52. Let the three charged particles, each of charge q, be located at the three corners of an equilateral triangle of side $a = 3$ cm in the horizontal plane. The distance of the charges from the centroid O is r. Apply law of cosines in $\triangle$OBC to get

$$r = a/\sqrt{3} = \sqrt{3}\ \text{cm}. \qquad (1)$$



Consider the electrostatic force on the charge at A due to the charges at B and C. The magnitude of forces due to each charge is

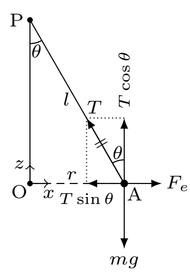
$$|\vec{F}| = \frac{q^2}{4\pi\epsilon_0 a^2}, \qquad (2)$$

and their directions are as shown in the figure. Resolve $\vec{F}$ along and perpendicular to AO. The resultant electrostatic force on charge at A is given by

$$\vec{F}_e = 2F\cos 30^\circ = \frac{\sqrt{3}q^2}{4\pi\epsilon_0 a^2}\,\hat{\imath}. \qquad (3)$$

The charges are connected with strings of length $l = 100$ cm. Let the other end of each string is hanging from the point P, which is vertically above O (see figure). In $\triangle$OAP,

$$\tan\theta = \frac{r}{\sqrt{l^2 - r^2}} = \frac{\sqrt{3}}{\sqrt{100^2 - 3}} = 0.017.$$



The forces on the charge at A are F_e, tension T, and weight mg (see figure). Resolve T in horizontal and vertical directions. The equilibrium condition on charge at A gives

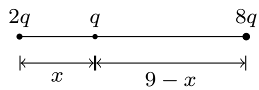
$$T\sin\theta = F_e = \frac{\sqrt{3}q^2}{4\pi\epsilon_0 a^2}, \qquad (4)$$

$$T\cos\theta = mg. \qquad (5)$$

Divide equation (4) by (5) and simplify to get

$$q = \left[\frac{4\pi\epsilon_0 a^2 mg\tan\theta}{\sqrt{3}}\right]^{1/2} = \left[\frac{(0.03)^2(10^{-3})(10)(0.017)}{9\times 10^9(1.73)}\right]^{1/2}$$
$$= 3.17\times 10^{-9}\ \text{C}.$$

53. The potential energy of two charges q_1 and q_2, separated by a distance r, is given by $U = q_1q_2/(4\pi\epsilon_0 r)$. For the potential energy of the given system to be minimum, the charges of the larger magnitude should be placed at the extreme positions (see figure). Let the charge q be placed at a distance x from the charge $2q$. The total potential energy of the given system is



$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0}\left[\frac{(2q)(8q)}{9}+\frac{(2q)(q)}{x}+\frac{(q)(8q)}{9-x}\right] \\ &= \frac{q^2}{2\pi\epsilon_0}\left[\frac{8}{9}+\frac{1}{x}+\frac{4}{9-x}\right]. \end{aligned} \qquad (1)$$

The value of x for which potential energy is minimum is given by $\mathrm{d}U/\mathrm{d}x = 0$ i.e.,

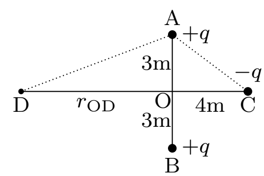
$$\frac{\mathrm{d}U}{\mathrm{d}x} = \frac{q^2}{2\pi\epsilon_0}\left[-\frac{1}{x^2}+\frac{4}{(9-x)^2}\right] = 0. \qquad (2)$$

Solve equation (2) to get $x = 3$. Note that another solution, $x = -9$, is not acceptable.
The electric field at the position of q due to the other two charges is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{x^2}-\frac{8q}{(9-x)^2}\right]\hat{\imath} = \frac{q}{4\pi\epsilon_0}\left[\frac{2}{(3)^2}-\frac{8}{(9-3)^2}\right]\hat{\imath} = \vec{0}.$$

Note that the force on the charge q is given by $\vec{F} = q\vec{E} = -\mathrm{d}U/\mathrm{d}x\,\hat{\imath}$. The charge q is placed at the position of a stable equilibrium.

54. Let the farthest point D be at a distance r_{OD} from O. The velocity of negative charge at D is $v_d = 0$ (because the charge changes its direction of motion at D). From geometry, $r_{\mathrm{AC}} = r_{\mathrm{BC}} = \sqrt{3^2+4^2} = 5$ m. The electrostatic potential energies of the negative charge at C and D are

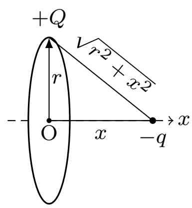


$$\begin{aligned} U_C &= -\frac{q^2}{4\pi\epsilon_0}\left[\frac{1}{r_{\mathrm{AC}}}+\frac{1}{r_{\mathrm{BC}}}\right] = -\frac{2(9\times10^9)(5\times10^{-5})^2}{5} = -9\ \mathrm{J}, \\ U_D &= -\frac{q^2}{4\pi\epsilon_0}\left[\frac{1}{r_{\mathrm{AD}}}+\frac{1}{r_{\mathrm{BD}}}\right] = -\frac{45}{\mathrm{r}_{\mathrm{AD}}}\ \mathrm{J}. \end{aligned}$$

The kinetic energy of the negative charge at C is $K_C = 4$ J and at D is $K_D = \frac{1}{2}mv_d^2 = 0$. Apply conservation of mechanical energy, $K_C + U_C = K_D + U_D$, to get $r_{\mathrm{AD}} = 45/5 = 9$ m. Pythagoras theorem in $\triangle$AOD gives

$$r_{\mathrm{OD}} = \sqrt{r_{\mathrm{AD}}^2 - r_{\mathrm{AO}}^2} = \sqrt{9^2-3^2} = \sqrt{72} = 8.48\ \mathrm{m}.$$

55. A positive charge $Q = 10^{-5}$ C is uniformly distributed over a ring of radius $r = 1$ m. The negative charge on the particle is $q = 10^{-6}$ C and its mass is $m = 0.9$ g. Let the particle be placed at a distance x from the centre of the ring. Apply Coulomb's law to show that the electrostatic force acting on the particle is



$$\vec{F} = -\frac{Qqx}{4\pi\epsilon_0(r^2+x^2)^{3/2}}\,\hat{\imath} = -\frac{Qqx}{4\pi\epsilon_0 r^3(1+x^2/r^2)^{3/2}}\,\hat{\imath}$$
$$\approx -\frac{Qq}{4\pi\epsilon_0 r^3}x\,\hat{\imath}. \qquad [\because (1+x^2/r^2)^{3/2} \approx 1 \text{ for } x \ll r]. \tag{1}$$

The restoring force $\vec{F}$ is proportional to the displacement from the centre of motion. Thus, the particle executes SHM. Apply Newton's second law on the particle to get its equation of motion

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{Qq}{4\pi\epsilon_0 m r^3}x = -\omega^2 x, \tag{2}$$

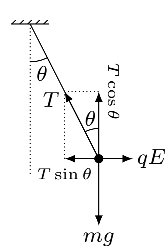
where ω is the angular frequency of SHM. The time period of oscillations is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4\pi\epsilon_0 m r^3}{Qq}}$$
$$= 2(3.14)\sqrt{\frac{(0.9\times10^{-3})\,(1)^3}{(9\times10^9)\,(10^{-5})\,(10^{-6})}} = 0.628 \text{ s}.$$

Note that the motion of the particle is periodic but not SHM, if $x \not\ll r$.

56. The electric force on a charge particle is tangential to the electric lines of forces. However, if the initial velocity of the particle makes an angle with the direction of force, the particle moves in a curved path e.g., particle moves perpendicular to the force in the uniform circular motion.

57. The ball of mass $m = 80$ mg and charge $q = 2\times10^{-8}$ C is placed in a horizontal uniform electric field $E = 20000$ V/m. Let T be the tension in the thread and θ be the angle it makes with the vertical (see figure). Resolve T in the horizontal and the vertical directions. In equilibrium, net force on the ball is zero i.e.,



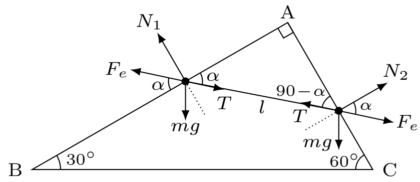
$$T\sin\theta = qE, \tag{1}$$
$$T\cos\theta = mg. \tag{2}$$

Solve equations (1) and (2) to get

$$\theta = \tan^{-1}\frac{qE}{mg} = \tan^{-1}\frac{(2\times10^{-8})(20000)}{(80\times10^{-6})(9.8)} = 27^\circ,$$

$$T = \sqrt{(qE)^2+(mg)^2} = \sqrt{(2\times10^{-8}\times20000)^2+(80\times10^{-6}\times9.8)^2}$$
$$= 8.8\times10^{-4}\ \text{N}.$$

58. The forces acting on the charge q_1 are its weight mg, normal reaction N_1, string tension T and Coulomb's force $F_e = q_1q_2/(4\pi\epsilon_0 l^2)$. Similarly, forces on the charge q_2 are its weight mg, normal reaction N_2, string tension T and Coulomb's force $F_e = q_1q_2/(4\pi\epsilon_0 l^2)$.



The forces are shown in the figure. In triangle ABC, $\angle A = 90^\circ$. Resolve the forces on q_1 and q_2 in the directions parallel and perpendicular to the sides of the frame. Balance the forces on q_1 to get

$$N_1 + F_e\sin\alpha = mg\cos30^\circ + T\sin\alpha, \quad (1)$$
$$F_e\cos\alpha + mg\sin30^\circ = T\cos\alpha, \quad (2)$$

and on q_2 to get

$$N_2 + F_e\cos\alpha = mg\cos60^\circ + T\cos\alpha, \quad (3)$$
$$F_e\sin\alpha + mg\sin60^\circ = T\sin\alpha. \quad (4)$$

Solve equations (2) and (4) to get $\alpha = 60^\circ$ and $T = mg + F_e$. Substitute these values in equations (1) and (3) to get $N_1 = \sqrt{3}mg$ and $N_2 = mg$. The tension becomes zero when the cord is cut. Substitute $T = 0$ to get $F_e = -mg$ i.e., $q_1q_2 = -4\pi\epsilon_0 l^2 mg$.