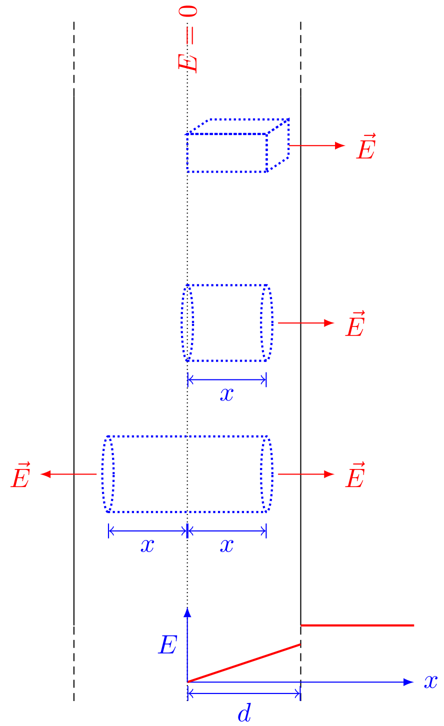
Problem: A nonconducting sheet of large surface area and thickness d contains uniform charge distribution of density ρ. Find the electric field at a point P inside the plate at a distance x from the central plane. Draw a qualitative graph of E agains x for $0 < x < d$.

Solution: Since surface area of the sheet is large, we can assume this to be an infinite sheet. By symmetry, the direction of the electric field inside (as well as outside) the sheet is perpendicular to its surface and is in outward direction (if ρ is positive). Also, the field at central plane should be zero.



Consider a cylinder of length x, cross-section area A $(= \pi r^2)$ and one of its face located on the central plane (see middle figure) as the Gaussian surface. The flux through the curved surface of this cylinder is zero because field is perpendicular to the area element on the curved surface i.e., $\mathrm{d}\phi = \vec{E} \cdot \mathrm{d}\vec{A} = |\vec{E}||\mathrm{d}\vec{A}| \cos 90° = 0$. The flux through the face on the central plane is zero because $E = 0$ on the central plane. The flux through the other face is $E_{\text{in}}A$, where E_{in} is the field inside the plate at a distance x from the central plane. Thus, total flux through this Gaussian surface is $\phi = E_{\text{in}}A$. The charge enclosed in this Gaussian surface is $q_{en} = \rho V = \rho A x$. Thus, by Gauss's law, $\phi = q_{en}/\epsilon$ i.e., $E_{\text{in}}A = \rho A x/\epsilon$ which gives $E_{\text{in}} = \rho x/\epsilon$ (here ϵ is the permittivity of the insulator).

For field outside the plate, consider a similar cylinder but of length $x > d$. The flux through this cylinder is $\phi = E_{\text{out}}A$ and charge enclosed in this is $q_{\text{en}} = \rho A d$. Apply Gauss's law to get $E_{\text{out}} = \rho d/\epsilon_0$. Note that the field outside is independent of x i.e., it is a constant. Here ϵ_0 is permittivity of the free space. Note that field is not continuous at $x = d$ (because $\epsilon \neq \epsilon_0$). Generally, field on the boundary of charged insulators is dis-continuous.

This problem can be solved using different Gaussian surfaces. The readers are encouraged to find the field by using other Gaussian surfaces shown in the figure.

© 2016 by Jitender Singh. Downloaded from www.concepts-of-physics.com 2016-05-27 22:28:33