IIT JEE PHYSICS
(1978–2016: 39 Years)
Topic-wise Complete Solutions

Volume I
Mechanics, Waves and Optics

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PsiPhiETC
2016

Typeset in TEX.

Second Edition, 2016 ⇝ 1.

ISBN 978-93-5265-609-7

Printed in India.

We dedicate this book to the hundreds of anonymous professors at IITs who formulated the challenging problems for IIT-JEE. The book is a showcase of their creation.

Foreword

Physics starts with observing the nature. The systematic observation results in simple rules which unlock the doors to the nature's mystery. Having learned a handful of simple rules, we can combine them logically to obtain more complicated rules and gain an insight into the way this world works. The skill, to apply the theoretical knowledge to solve any practical problem, comes with regular practice of solving problems. The aim of the present collection of problems and solutions is to develop this skill.

IIT JEE questions had been a challenge and a center of attraction for a big section of students at intermediate and college level. Independent of their occurrence as an evaluation tool, they have good potential to open up thinking threads in mind. Jitender Singh and Shraddhesh Chaturvedi have used these questions to come up with a teaching material that can benefit students. The explanations accompanying the problems could bring conceptual clarity and develop the skills to approach any unseen problem, step by step. These problems are arranged in a chapter sequence that is used in my book Concepts of Physics. Thus a student using both the books will find it as an additional asset.

Both Jitender Singh and Shraddhesh Chaturvedi have actually been my students at IIT, Kanpur. Jitender Singh has been closely associated with me since long. It gives me immense pleasure to see that my own students are furthering the cause of Physics education. I wish them every success in this work and expect much more contribution from them in future!

Dr. H C Verma
Professor of Physics
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Preface

This book provides a comprehensive collection of IIT JEE problems and their solutions. We have tried to keep our explanations simple so that any reader, with basic knowledge of intermediate physics, can understand them on his/her own without any external assistance. It can be, therefore, used for self-study.

To us, every problem in this book, is a valuable resource to unravel a deeper understanding of the underlying physical concepts. The time required to solve a problem is immaterial as far as Physics is concerned. We believe that getting the right answer is often not as important as the process followed to arrive at it. The emphasis in this text remains on the correct understanding of the principles of Physics and on their application to find the solution of the problems. If a student seriously attempts all the problems in this book, he/she will naturally develop the ability to analyze and solve complex problems in a simple and logical manner using a few, well-understood principles.

For the convenience of the students, we have arranged the problems according to the standard intermediate physics textbook. Some problems might be based on the concepts explained in multiple chapters. These questions are placed in a later chapter so that the student can try to solve them by using the concept(s) from multiple chapters. This book can, thus, easily complement your favorite text book as an advanced problem book.

The IIT JEE problems fall into one of the nine categories: (i) MCQ with single correct answer (ii) MCQ with one or more correct answers (iii) Paragraph based (iv) Assertion Reasoning based (v) Matrix matching (vi) True False type (vii) Fill in the blanks (viii) Integer Type, and (ix) Subjective. Each chapter has sections according to these categories. In each section, the questions are arranged in the descending order of year of appearance in IIT JEE.

The solutions are given at the end of each chapter. If you can't solve a problem, you can always look at the solution. However, trying it first will help you identify the critical points in the problems, which in turn, will accelerate the learning process. Furthermore, it is advised that even if you think that you know the answer to a problem, you should turn to its solution and check it out, just to make sure you get all the critical points.

This book has a companion website, `www.concepts-of-physics.com`. The site will host latest version of the errata list and other useful material. We would be glad to hear from you for any suggestions on the improvement of the book. We have tried our best to keep the errors to a minimum. However, they might still remain! So, if you find any conceptual errors or typographical errors, howsoever small and insignificant, please inform us so that it can be corrected in the later editions. We believe, only a collaborative effort from the students and the authors can make this book absolutely error-free, so please contribute.

Many friends and colleagues have contributed greatly to the quality of this book. First and foremost, we thank Dr. H. C. Verma, who was the inspiring force behind this project. Our close friends and classmates from IIT Kanpur, Deepak Sharma, Chandrashekhar Kumar and Akash Anand stood beside us throughout this work. This work would not have been possible without the constant support of our wives Reena and Nandini and children Akshaj, Viraj and Maitreyi.

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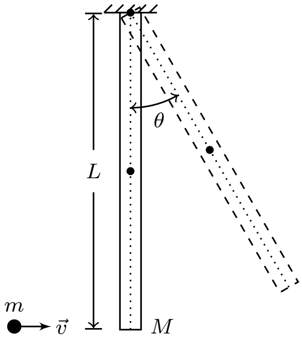
Part I

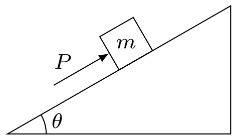
Mechanics

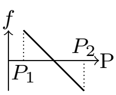


Friction

One Option Correct

1. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f *versus* P graph will look like (2010)



(A)


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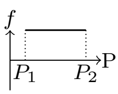

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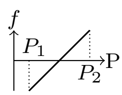

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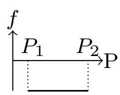
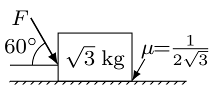
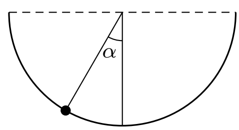


2. What is the maximum value of the force F such that the block shown in the arrangement does not move? [Take $g = 10\ \mathrm{m/s^2}$.] (2003)



(A) 20 N (B) 10 N (C) 12 N (D) 15 N

3. An insect crawls up a hemispherical surface very slowly (see figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by (2001)



(A) $\cot\alpha = 3$ (B) $\tan\alpha = 3$ (C) $\sec\alpha = 3$ (D) $\operatorname{cosec}\alpha = 3$

4. A block of mass 0.1 kg is held against a wall by applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is (1994)

(A) 2.5 N (B) 0.98 N (C) 4.9 N (D) 0.49 N

5. If a machine is lubricated with oil, (1980)

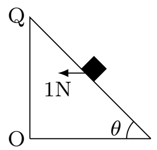
(A) the mechanical advantage of the machine increases.

(B) the mechanical efficiency of the machine increases.
(C) both its mechanical advantage and mechanical efficiency increases.
(D) its efficiency increases, but its mechanical advantage decreases.

6. A block of mass 2 kg rests on a rough inclined plane making an angle of $30°$ with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is (1980)
(A) 9.8 N (B) $0.7 \times 9.8 \times \sqrt{3}$ N (C) $9.8 \times \sqrt{3}$ N (D) 0.7×9.8 N

One or More Option(s) Correct

7. A small block of mass 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if [Take $g = 10$ m/s^2.] (2012)



(A) $\theta = 45°$
(B) $\theta > 45°$ and frictional force acts on the block towards P
(C) $\theta > 45°$ and frictional force acts on the block towards Q
(D) $\theta < 45°$ and frictional force acts on the block towards Q

Assertion Reasoning Type

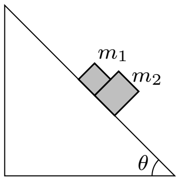
8. *Statement 1:* It is easier to pull a heavy object than to push it on a level ground.

Statement 2: The magnitude of frictional force depends on the nature of the two surfaces in contact. (2008)
(A) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
(B) Statement 1 is true, statement 2 is true; statement 2 is *not* a correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

9. *Statement 1:* A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

Statement 2: For every action there is an equal and opposite reaction. (2007)
(A) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
(B) Statement 1 is true, statement 2 is true; statement 2 is *not* a correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

Matrix or Matching Type



10. A block of mass $m_1 = 1$ kg and another block of mass $m_2 = 2$ kg are placed together on an inclined plane with angle of inclination θ (see figure). Various values of θ are given in *Column I*. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In *Column II* expressions for the friction on block m_2 are given. Match the correct expression of the friction in *Column II* with the angles given in *Column I*. The acceleration due to gravity is denoted by g. [Given, $\tan(5.5°) \approx 0.1$, $\tan(11.5°) \approx 0.2$, $\tan(16.5°) \approx 0.3$]. (2014)

Column I	Column II
(P) $\theta = 5°$	(1) $m_2 g \sin\theta$
(Q) $\theta = 10°$	(2) $(m_1 + m_2) g \sin\theta$
(R) $\theta = 15°$	(3) $\mu m_2 g \cos\theta$
(S) $\theta = 20°$	(4) $\mu (m_1 + m_2) g \cos\theta$

True False Type

11. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. (1981)

Fill in the Blank Type

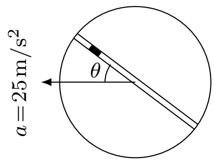
12. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is ...... N. (1984)

Integer Type

13. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is three times the force required to just prevent it from sliding down. If we define $N = 10\mu$ then N is .... (2011)

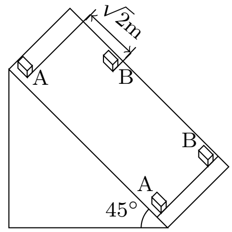
Descriptive

14. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown in the figure. The coefficient of friction between the block and all surfaces of groove in contact is $\mu = 2/5$. The disc has an acceleration of $25\ \mathrm{m/s^2}$. Find the acceleration of the block with respect to disc. (2006)

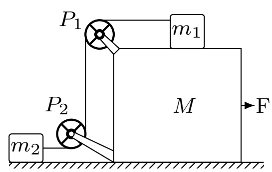


$\cos\theta = 4/5,\ \sin\theta = 3/5$

15. Two blocks A and B of equal masses are released from an inclined plane of inclination 45° at $t = 0$. Both the blocks are initially at rest. The coefficient of kinetic friction between the block A and the inclined plane is 0.2 while it is 0.3 for the block B. Initially the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line. [Take $g = 10\ \mathrm{m/s^2}$.] (2004)



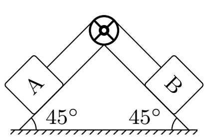
16. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and between m_2 and ground is 0.3. The pulleys and the strings are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to the mass M. [Take $g = 10\ \mathrm{m/s^2}$.] (2000)



(a) Draw a free body diagram of mass M, clearly showing all the forces.

(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular force F it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write equations of motion of all the masses. Find F, tension in the string and acceleration of the masses.

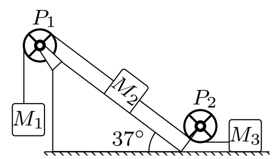
17. Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless, inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the blocks A and B are released from rest, find, (1997)



(a) the acceleration of A.
(b) tension in the string.
(c) the magnitude and direction of friction force acting on A.

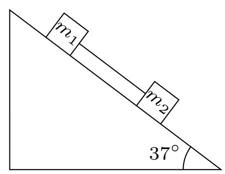
18. A block of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied. (1987)

19. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which passes over massless and frictionless pulleys P_1 and P_2 as shown in figure. The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surface is 0.25. The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity, find, [Take $g = 9.8\ \mathrm{m/s^2}$, $\sin 37° \approx 3/5$.] (1981)

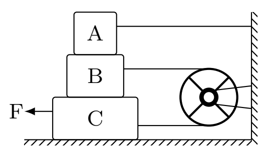


(a) the mass of M_1.
(b) the tension in the horizontal portion of the string.

20. Two blocks connected by a massless string slides down an inclined plane having an angle of inclination of 37°. The masses of the two blocks are $m_1 = 4$ kg and $m_2 = 2$ kg respectively and the coefficients of friction of m_1 and m_2 with the inclined plane are 0.75 and 0.25 respectively. Assuming the string to be taut, find (a) the common acceleration of two masses, and (b) the tension in the string. [Take $\sin 37° = 0.6$, $\cos 37° = 0.8$, $g = 9.8\ \mathrm{m/s^2}$.] (1979)



21. In the figure, the blocks A, B and C have masses 3 kg, 4 kg and 8 kg respectively. The coefficient of sliding friction between any two surfaces is 0.25. A is held at rest by a massless rigid rod fixed to the wall, while B and C are connected by a light flexible cord passing around a fixed frictionless pulley. Find the force F necessary to drag C along the horizontal surface to the left at a constant speed. Assume that the arrangement shown in the figure *i.e.*, B on C and A on B, is maintained throughout. [Take $g = 10\ \mathrm{m/s^2}$.] (1978)



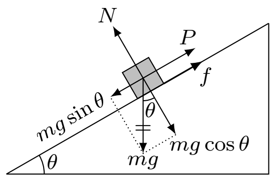
22. A block of mass 2 kg slides on an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\sqrt{3}/\sqrt{2}$. What force along the plane should be applied to the block so that it moves (a) down, and (b) up, without any acceleration? [Take $g = 10\ \mathrm{m/s^2}$.] (1978)

Answers

1. A
2. A
3. A
4. B
5. B
6. A
7. A, C
8. B
9. B
10. P$\mapsto$2, Q$\mapsto$2, R$\mapsto$3, S$\mapsto$3
11. F
12. 5
13. 5
14. 10 m/s^2
15. $S_A = 8\sqrt{2}$ m, 2 s
16. (b) $f_1 = 30$ N, $f_2 = 15$ N, $F = 60$ N, $T = 18$ N, $a = \frac{3}{5}$ m/s^2
17. (a) 0 (b) $\frac{2\sqrt{2}}{3}mg$ (c) $\frac{mg}{3\sqrt{2}}$, downwards
18. $\frac{\mu mg}{\sqrt{1+\mu^2}}$, $\tan^{-1}\mu$
19. (a) 4.2 kg (b) 9.8 N
20. (a) 1.3 m/s^2 (b) 5.2 N
21. 80 N
22. (a) 11.21 N (b) 31.21 N

Solutions

1. The forces acting on the block are its weight mg, normal reaction N, applied force P and frictional force f (see figure). Resolve mg along and normal to the plane and apply Newton's second law to get



$$0 = P + f - mg\sin\theta,$$

which gives

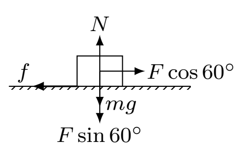
$$f = -P + mg\sin\theta. \tag{1}$$

This is a straight line with slope -1. Substitute the values of P_1 and P_2 in equation (1) to get the frictional force at these points i.e.,

$$f_1 = \mu mg\cos\theta, \qquad \text{and} \qquad f_2 = -\mu mg\cos\theta.$$

The readers are encouraged to show that if $P < P_1$ then the block starts sliding down and if $P > P_2$ then block starts moving up. Note that $P_1 > 0$ because $\tan\theta > \mu$.

2. The forces acting of the block are applied force F, weight mg, normal reaction N, and the frictional force f as shown in the figure. Resolve F in the horizontal and the vertical directions and apply Newton's second law to get



$$N = F\sin 60° + mg, \tag{1}$$
$$f = F\cos 60°. \tag{2}$$

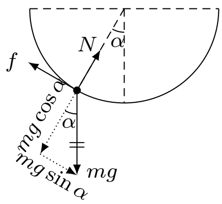
The force F becomes maximum when the friction force f attains its maximum value i.e.,

$$f = \mu N. \tag{3}$$

Eliminate f and N from equations (1)–(3) to get

$$F_{\max} = \frac{\mu m g}{\cos 60^\circ - \mu \sin 60^\circ} = 20\ \text{N}.$$

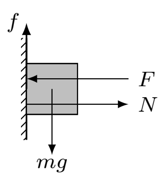
3. The forces on the insect are its weight mg, normal reaction N, and the frictional force f (see figure). Resolve mg along and perpendicular to the normal. Apply equilibrium condition to get



$$f = mg\sin\alpha, \tag{1}$$
$$N = mg\cos\alpha. \tag{2}$$

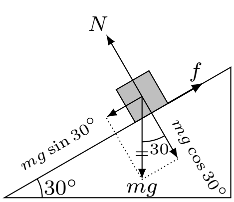
The angle α attains its maximum value when the frictional force reaches its maximum limit of $f_{\max} = \mu N$. Substitute in equation (1) and then divide by equation (2) to get $\tan\alpha = \mu = 1/3$. Thus, $\cot\alpha = 3$.

4. The forces on the block are applied force $F = 5$ N, normal reaction N, weight mg and the frictional force f. In equilibrium, $N = F = 5$ N and $f = mg = 0.1 \times 9.8 = 0.98$ N. Note that f is less than its maximum possible value of $f_{\max} = \mu N = 0.5 \times 5 = 2.5$ N.



5. Lubrication reduces the non-conservative frictional forces. This increases the efficiency of the machine.

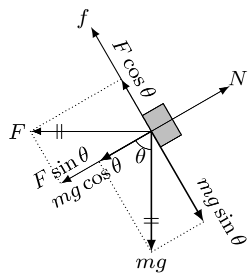
6. The forces on the block of mass $m = 2$ kg are its weight mg, normal reaction N, and the frictional force f (see figure). The net force on the block is zero because it is at rest. Resolve mg in the directions parallel and perpendicular to the inclined plane. Apply Newton's second law in these directions to get



$$N = mg\cos 30^\circ = (2)(9.8)(0.866) = 16.97\ \text{N}, \tag{1}$$
$$f = mg\sin 30^\circ = (2)(9.8)(0.5) = 9.8\ \text{N}. \tag{2}$$

Note that f is less than $f_{\max} = \mu N = (0.7)(16.97) = 11.88$ N.

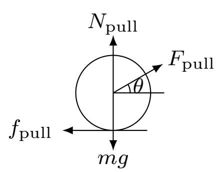
7. The forces acting on the block are $F = 1$ N towards the left, weight $mg = 0.1 \times 10 = 1$ N downwards, normal force N, and the frictional force f. Resolve F and mg along and perpendicular to the plane (see figure). When $\theta = 45^\circ$, the net force that brings the block down is



$$F_\mathrm{d} = mg\sin\theta - F\cos\theta = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0.$$

Thus, the block is stationary if $\theta = 45^\circ$. When $\theta > 45^\circ$, the force $F_d > 0$, and hence the block has a tendency to move down. Thus, the frictional force f acts on the block upwards i.e., towards Q.

8. Both, statement 1 and statement 2, are true but statement 2 is not a correct explanation of statement 1. The forces acting on the body in pull case are shown in the figure. In this case, the body will just start moving when the horizontal component of F_pull is equal to or greater than the maximum value of friction force i.e.,



$$F_\mathrm{pull}\cos\theta = f_\mathrm{pull} = \mu N_\mathrm{pull}. \qquad (1)$$

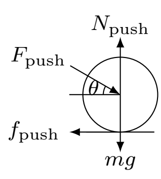
Since there is no acceleration in the vertical direction

$$N_\mathrm{pull} = mg - F_\mathrm{pull}\sin\theta. \qquad (2)$$

Eliminate N_pull from equations (1) and (2) to get

$$F_\mathrm{pull} = \frac{\mu mg}{\cos\theta + \mu\sin\theta}.$$

The forces acting on the body in push case are shown in the figure. In this case,



$$F_\mathrm{push}\cos\theta = f_\mathrm{push} = \mu N_\mathrm{push},$$
$$N_\mathrm{push} = mg + F_\mathrm{push}\sin\theta,$$
$$F_\mathrm{push} = \frac{\mu mg}{\cos\theta - \mu\sin\theta}.$$

Note that $F_\mathrm{pull} = F_\mathrm{push} = \mu mg$ if $\theta = 0$.

9. Both statements are true but statement 2 is not a correct explanation for statement 1. Generally, statement 1 is attributed to Newton's first law but this is not entirely correct. The readers are encouraged to repeat this experiment by pulling the cloth slowly. The outcome of experiment depends on acceleration a_cloth of the cloth (see

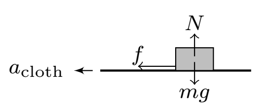


figure). The forces acting on the dish are its weight mg, normal reaction N, and the frictional force f. Maximum value of the frictional force is

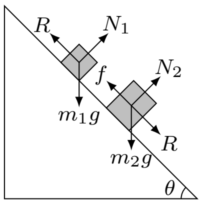
$$f_{\max} = \mu N = \mu mg. \tag{1}$$

By Newton's second law, acceleration of the dish is $a_{\text{dish}} = f/m$. If a_{cloth} is small then the cloth and the dish move together i.e., $a_{\text{dish}} = a_{\text{cloth}}$. Maximum value of a_{cloth} for the cloth and the dish to move together is given by

$$a_{\text{cloth}} = a_{\text{dish}} = f_{\max}/m = \mu g. \tag{2}$$

Beyond this limit, $a_{\text{cloth}} > a_{\text{dish}} = \mu g$ and hence the cloth comes out leaving the dish on table.

10. The forces on the block of mass m_1 are its weight $m_1 g$, normal reaction from the inclined plane N_1, and the reaction from the second block R (see figure). Similarly, forces on the block of mass m_2 are $m_2 g$, N_2, R, and the frictional force f. If θ is slowly increased, f starts increasing and attains its maximum value $f = \mu N_2$ at $\theta = \theta_r$ (angle of repose). The blocks are stationary if $\theta \leq \theta_r$ otherwise they are moving. Consider the limiting case, $\theta = \theta_r$, when the blocks are at rest and



$$f = \mu N_2. \tag{1}$$

Apply Newton's second law to m_1,

$$R = m_1 g \sin\theta, \tag{2}$$
$$N_1 = m_1 g \cos\theta, \tag{3}$$

and to m_2,

$$f = R + m_2 g \sin\theta, \tag{4}$$
$$N_2 = m_2 g \cos\theta. \tag{5}$$

Eliminate R, N_2, and f from equations (1)–(5) to get

$$\theta_r = \tan^{-1}\left(\frac{\mu m_2}{m_1 + m_2}\right) = \tan^{-1}\left(\frac{0.3 \times 2}{1+2}\right) = \tan^{-1}(0.2) = 11.5°.$$

Thus, for $\theta = 5°$ and $\theta = 10°$, blocks are at rest with frictional force

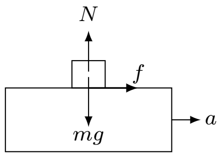
$$f = R + m_2 g \sin\theta = (m_1 + m_2) g \sin\theta.$$

For $\theta = 15°$ and $\theta = 20°$, blocks are moving with frictional force

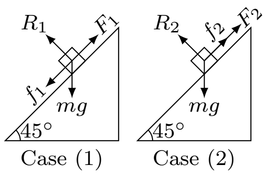
$$f = \mu N_2 = \mu m_2 g \cos\theta, \qquad \text{(limiting value)}.$$

11. To walk in the forward direction, the person pushes his foot backward. Thus, the foot has a tendency to move backward against the rough surface. To oppose this movement, the friction force on the foot acts in the forward direction. Note that the frictional force is the only horizontal force on the person. Thus, a person can accelerate in forward direction only if the frictional force is forward.

12. The acceleration of the block is equal to the acceleration of the truck i.e., $a = 5\ \mathrm{m/s^2}$. Only horizontal force acting on the block is frictional force f. Apply Newton's second law to get $f = ma = (1)(5) = 5$ N. Note that the limiting value of frictional force is $f_{\max} = \mu N = \mu mg = 0.6(1)(9.8) = 5.88$ N.



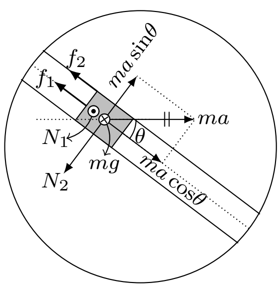
13. Suppose block just slides up in *case 1* and just slides down in *case 2*. The forces acting on the block are weight (mg), normal reaction (R), applied force (F_1 in *case 1* and F_2 in *case 2*) and frictional force (f_1 in *case 1* and f_2 in *case 2*), as shown in the figure. The equilibrium conditions in *case 1* and *case 2* give



$$R_1 = mg\cos 45^\circ, \qquad F_1 = mg\sin 45^\circ + f_1,$$
$$R_2 = mg\cos 45^\circ, \qquad F_2 = mg\sin 45^\circ - f_2.$$

Using $F_1 = 3F_2$, $f_1 = \mu R_1$, and $f_2 = \mu R_2$, we get $\mu = 0.5$. Thus, $N = 5$.

14. Let us solve the problem in a frame attached to the disc. This frame is accelerating towards the left with acceleration $a = 25\ \mathrm{m/s^2}$. We can apply Newton's second law in this non-inertial frame if we apply a psedo-force ma acting towards the right. The forces acting on the block are pseudo force ma towards the right, its weight mg into the paper, normal reaction on the bottom surface of the block N_1 coming out of the paper, frictional force



$$f_1 = \mu N_1, \tag{1}$$

corresponding to N_1, normal reaction on the side surfaces of the block N_2 and frictional force

$$f_2 = \mu N_2, \tag{2}$$

corresponding to N_2 (see figure). The constraint that the block moves in a horizontal plane along the groove gives

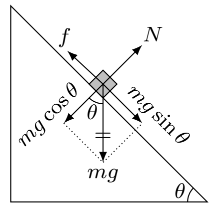
$$N_1 = mg, \tag{3}$$
$$N_2 = ma\sin\theta. \tag{4}$$

Apply Newton's second law along the groove to get

$$ma\cos\theta - f_1 - f_2 = ma_r. \tag{5}$$

Substitute the values of the parameters to get $a_r = 10\ \mathrm{m/s^2}$.

15. The forces acting on the blocks are weight mg, normal reaction N and the frictional force $f = \mu N$. Newton's second law gives



$$\begin{aligned} N &= mg\cos\theta, \\ ma &= mg\sin\theta - f = mg\sin\theta - \mu N \\ &= mg\sin\theta - \mu mg\cos\theta. \end{aligned}$$

Thus, acceleration of the two blocks are

$$a_A = g\sin\theta - \mu_A g\cos\theta = 8/\sqrt{2}, \tag{1}$$

$$a_B = g\sin\theta - \mu_B g\cos\theta = 7/\sqrt{2}, \tag{2}$$

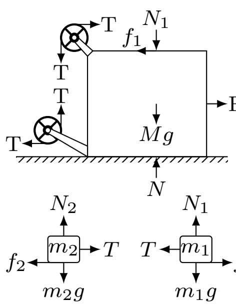
where $\mu_A = 0.2$, $\mu_B = 0.3$ and $\theta = 45°$. Let front faces comes in a line at time t. The distances travelled by A and B in time t are

$$S_A = \tfrac{1}{2}\,a_A t^2 = (4/\sqrt{2})t^2, \tag{3}$$

$$S_B = \tfrac{1}{2}\,a_B t^2 = (3.5/\sqrt{2})t^2. \tag{4}$$

Using $S_A - S_B = \sqrt{2}$, we get $t = 2$ s and $S_A = 8\sqrt{2}$ m.

16. Let T be the tension in the string, N be the reaction force between M and ground, N_1 be the reaction force between m_1 and M, f_1 be the friction force on m_1, and f_2 be the friction force on m_2. The free body diagrams of m_1, m_2, and M are shown in the figure. Apply Newton's second law on m_1, m_2, and M in the vertical direction to get



$$N_1 = m_1 g, \quad N_2 = m_2 g, \quad N = N_1 + Mg.$$

Limiting (maximum) values of the frictional forces are

$$f_{1,\max} = \mu N_1 = \mu m_1 g = 0.3 \times 20 \times 10 = 60\ \mathrm{N},$$

$$f_{2,\max} = \mu N_2 = \mu m_2 g = 0.3 \times 5 \times 10 = 15\ \mathrm{N}.$$

Let a_1, a_2, and A be the rightward accelerations of m_1, m_2, and M w.r.t. the ground. As string is inextensible, the accelerations of m_1 and m_2 are equal i.e., $a_1 = a_2 = a$. Apply Newton's second law on m_1, m_2, and M in the horizontal direction to get

$$T - f_2 = m_2 a_2 = m_2 a, \tag{1}$$

$$f_1 - T = m_1 a_1 = m_1 a, \tag{2}$$

$$F - f_1 = MA. \tag{3}$$

The magnitude of F decides the values of the frictional forces f_1 and f_2 and the motion of the blocks. There are two possible cases

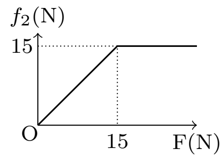
(i) m_1 does not slide over M (i.e., $A = a$), and

(ii) m_1 slides over M (i.e., $a \neq A$) and hence $f_1 = f_{1,\text{max}}$.

Consider the horizontal motion of the blocks in case (i). In this case, the external forces on the system of m_1, m_2, and M are F and f_2. The system starts moving only when $F > f_{2,\text{max}}$.

The f_2 adjusts itself such that it is equal to F till it reaches the maximum value of $f_{2,\text{max}}$ i.e.,

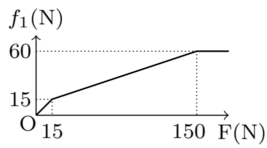
$$f_2 = \begin{cases} F, & \text{if } F \leq 15 \text{ N}; \\ 15 \text{ N}, & \text{otherwise.} \end{cases} \tag{4}$$



Eliminate T and A from equations (1)–(3) to get the frictional force

$$f_1 = \frac{f_2 M + (m_1 + m_2)F}{m_1 + m_2 + M}. \tag{5}$$

From equations (4) and (5), the f_1 is equal to F till $F \leq 15$ N, then it increases linearly with F with a slope $\frac{m_1+m_2}{m_1+m_2+M}$ till it reaches a value of $f_{1,\text{max}} = 60$ N, and then remains constant at its limiting value (see figure). Mathematically,



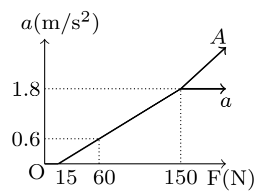
$$f_1 = \begin{cases} F, & \text{if } F \leq 15 \text{ N}; \\ \frac{15M+(m_1+m_2)F}{m_1+m_2+M}, & \text{if } 15 \text{ N} < F \leq 150 \text{ N}; \\ 60 \text{ N}, & \text{otherwise.} \end{cases} \tag{6}$$

Note that $f_1 = 60$ N when $F = 150$ N.

Eliminate T from equations (1) and (2) to get the acceleration

$$a = (f_1 - f_2)/(m_1 + m_2). \tag{7}$$

Substitute f_2 and f_1 from equations (5) and (6) in equation (7) to get

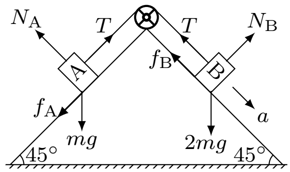


$$a = \begin{cases} 0, & \text{if } F < 15 \text{ N}; \\ \frac{F-15}{m_1+m_2+M} = \frac{F-15}{75}, & \text{if } 15 \text{ N} < F \leq 150 \text{ N}; \\ 1.8 & \text{otherwise.} \end{cases} \tag{8}$$

Beyond $F = 150$ N, the assumption that m_1 does not slide over M is invalid and case (ii) comes into play. In this case, the acceleration a remains at its maximum value of 1.8 m/s^2. Using equation (3), the acceleration A varies with F as

$$A = (F - f_{1,\text{max}})/M = (F - 60)/50. \tag{9}$$

From equations (5) and (6), the condition $f_1 = 2f_2$ occurs when $f_2 = f_{2,\text{max}} = 15$ N and $f_1 = 2f_{2,\text{max}} = 30$ N. Substitute the values of f_1 and f_2 to get $a = 0.6\ \text{m/s}^2$, $F = 60$ N, and $T = 18$ N.



17. Let block B is accelerating down the plane with an acceleration a. The string is inextensible which makes block A to move up the plane with the same acceleration a. The tension T in the string remains same throughout the string as it is massless and the pulley is frictionless. The forces on the two blocks are shown in the figure. Resolve forces along and normal to the plane. Apply Newton's second law in normal direction to get

$$N_\text{A} = mg\cos 45^\circ = mg/\sqrt{2}, \tag{1}$$
$$N_\text{B} = 2mg\cos 45^\circ = \sqrt{2}\,mg. \tag{2}$$

Apply Newton's second law along the plane to get

$$T - mg/\sqrt{2} - f_\text{A} = ma, \tag{3}$$
$$\sqrt{2}\,mg - T - f_\text{B} = 2ma. \tag{4}$$

Eliminate T from equations (3) and (4) to get

$$a = \frac{1}{3m}\left(mg/\sqrt{2} - f_\text{A} - f_\text{B}\right). \tag{5}$$

When blocks move relative to the plane then frictional forces attain their maximum values which are given by

$$f_\text{A} = f_{\text{A},\text{max}} = \mu_\text{A} N_\text{A} = (2/3)(mg/\sqrt{2}) = \sqrt{2}mg/3, \tag{6}$$
$$f_\text{B} = f_{\text{B},\text{max}} = \mu_\text{B} N_\text{B} = (1/3)(\sqrt{2}\,mg) = \sqrt{2}mg/3. \tag{7}$$

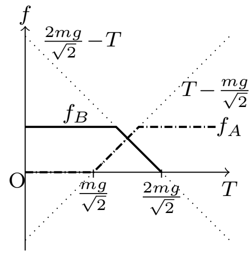
Substitute these values in equation (5) to get $a = -g/9\sqrt{2}$. The negative sign shows that our assumption of block B moving down the plane (and hence assumed directions of f_A and f_B) was wrong. If we assume block B to move up and rewrite equations (3) and (4), we get

$$a = \frac{1}{3m}\left(-mg/\sqrt{2} - f_\text{A} - f_\text{B}\right), \tag{8}$$

again a negative quantity.

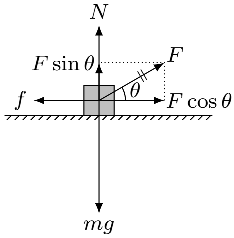
Thus, blocks are neither moving up or down i.e., $a = 0$. Substitute $a = 0$ in equations (3) and (4) to get

$$f_{\mathrm{A}} = T - mg/\sqrt{2},$$
$$f_{\mathrm{B}} = 2mg/\sqrt{2} - T.$$



The minimum and the maximum values of f_{A} and f_{B} are zero and $\sqrt{2}\,mg/3$. The variation of f_A and f_B with tension T is shown in the figure. It can be seen that f_B attains its maximum value earlier than f_A attains its maximum. Substitute $f_B = \sqrt{2}\,mg/3$, to get $T = 2\sqrt{2}\,mg/3$ and $f_{\mathrm{A}} = \sqrt{2}\,mg/3$.

18. Let the force F be applied at an angle θ to the horizontal plane. The forces acting on the block of mass m are its weight mg, normal reaction N, frictional force f, and the applied force F (see figure). Resolve $\vec{F}$ in the horizontal and the vertical directions. The net force in the vertical direction is zero (because there is no acceleration in the vertical direction) i.e.,



$$N + F\sin\theta = mg. \tag{1}$$

The block will just start moving when the horizontal component of the applied force is equal to the limiting value of the frictional force i.e.,

$$F\cos\theta = f_{\mathrm{max}} = \mu N. \tag{2}$$

Eliminate N from equations (1) and (2) and simplify to get

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}. \tag{3}$$

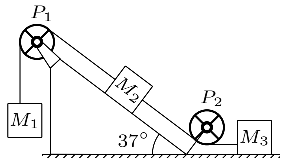
The applied force is minimum when $\mathrm{d}F/\mathrm{d}\theta = 0$ i.e.,

$$\frac{\mathrm{d}F}{\mathrm{d}\theta} = -\frac{\mu mg}{(\cos\theta + \mu\sin\theta)^2}(-\sin\theta + \mu\cos\theta) = 0, \tag{4}$$

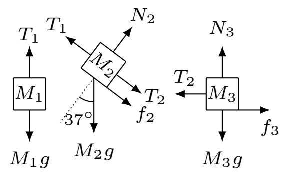
which gives $\theta = \tan^{-1}\mu$. Substitute θ in equation (3) to get

$$F_{\mathrm{min}} = \frac{\mu mg}{\sqrt{1+\mu^2}}.$$

The readers are encouraged to solve this problem if the block rests on an inclined plane.



19. The masses M_1, M_2, and M_3 are connected by the inextensible strings. Since M_1 moves downwards with a uniform velocity, all the three masses will moves with the same uniform velocity because they are connected by the inextensible strings. Thus, accelerations of M_1, M_2, and M_3 are zero. By Newton's second law, the net force on each mass is zero.

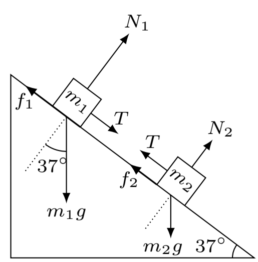


Let T_1 be the tension in the string that connects M_1 and M_2 and T_2 be the tension in the string that connects M_2 and M_3. The normal reactions on M_2 and M_3 are N_2 and N_3, and the frictional forces on these blocks are f_2 and f_3. The free body diagrams of the three masses are shown in the figure. Resolve M_2g along and normal to the inclined plane. The frictional force on the mass M_2 is $f_2 = \mu N_2$ and that on the mass M_3 is $f_3 = \mu N_3$ because these blocks are sliding on the rough surfaces with the coefficient of kinetic friction $\mu = 0.25$. Apply Newton's second law on the three blocks to get

$$
\begin{aligned}
T_1 &= M_1 g, && (1)\\
N_2 &= M_2 g \cos 37°, && (2)\\
T_1 &= T_2 + \mu N_2 + M_2 g \sin 37°, \qquad (\because f_2 = \mu N_2) && (3)\\
N_3 &= M_3 g, && (4)\\
T_2 &= \mu N_3. \qquad (\because f_3 = \mu N_3). && (5)
\end{aligned}
$$

Solve equations (1)–(5) to get

$$
\begin{aligned}
T_2 &= \mu M_3 g = (0.25)(4)(9.8) = 9.8\ \text{N},\\
M_1 &= \mu M_3 + \mu M_2 \cos 37° + M_2 \sin 37°\\
&= (0.25)(4) + (0.25)(4)(4/5) + (4)(3/5) = 21/5\ \text{kg}.
\end{aligned}
$$



20. The forces acting on the block of mass $m_1 = 4$ kg are its weight $m_1 g$, normal reaction N_1, frictional force f_1, and the string tension T (see figure). Similarly, forces on the block of mass $m_2 = 2$ kg are its weight $m_2 g$, normal reaction N_2, frictional force f_2, and the string tension T. Resolve $m_1 g$ and $m_2 g$ in directions parallel and perpendicular to the incline. Apply Newton's second law on m_1 and m_2 in a direction perpendicular to the incline to get

$$
\begin{aligned}
N_1 &= m_1 g \cos 37°, && (1)\\
N_2 &= m_2 g \cos 37°. && (2)
\end{aligned}
$$

The frictional forces on the sliding blocks m_1 and m_2 are equal to their limiting values

$$f_1 = \mu_1 N_1 = \mu_1 m_1 g \cos 37°, \tag{3}$$
$$f_2 = \mu_2 N_2 = \mu_2 m_2 g \cos 37°, \tag{4}$$

where $\mu_1 = 0.75$ and $\mu_2 = 0.25$ are the coefficients of friction for m_1 and m_2. Let blocks are accelerating down with an equal acceleration a (note that the string remains taut). Apply Newton's second law on m_1 and m_2 in a direction parallel to the plane to get

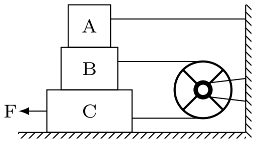
$$m_1 g \sin 37° + T - f_1 = m_1 a, \tag{5}$$
$$m_2 g \sin 37° - T - f_2 = m_2 a. \tag{6}$$

Add equations (5) and (6) and substitute f_1 and f_2 from equations (3) and (4) to get

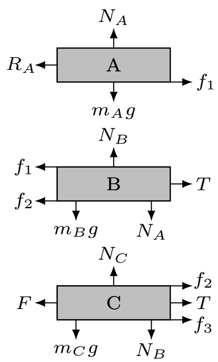
$$\begin{aligned} a &= g \sin 37° - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2} g \cos 37°, \\ &= (9.8)(0.6) - \frac{(0.75)(4) + (0.25)(2)}{4+2}(9.8)(0.8) = 1.3\ \mathrm{m/s^2}. \end{aligned}$$

Now, multiply equation (5) by m_2 and equation (6) by m_1 and then subtract to get,

$$\begin{aligned} T &= \frac{(\mu_1 - \mu_2) m_1 m_2}{m_1 + m_2}\, g \cos 37°, \\ &= \frac{(0.75 - 0.25)(4)(2)}{4+2}\,(9.8)(0.8) = 5.2\ \mathrm{N}. \end{aligned}$$



21. The masses of the blocks are $m_A = 3$ kg, $m_B = 4$ kg, and $m_C = 8$ kg. The coefficient of sliding friction between any two surfaces is $\mu = 0.25$. Let the frictional force between the block A and the block B be f_1, between the block B and the block C be f_2, and between the block C and the ground be f_3. The applied force F slides the block C along the horizontal surface to the left at a constant speed. The frictional force f_3 on the block C resists this movement by acting in the rightward direction. Since the string is inextensible and the pulley is fixed, the block B starts moving towards the right with a speed equal to that of C. The frictional force f_2 on B resists this motion by acting in the leftward direction. The frictional force f_2 on the block C acts in the rightward direction (Newton's third law). The block A is fixed and the block B moves towards the right relative to the block A. The frictional force f_1 on the block B acts in the leftward direction to oppose this motion. The force f_1 on the block A acts in the rightward direction.



The free body diagrams of the blocks A, B, and C are shown in the figure. The forces on the block A are its weight m_Ag, normal reaction N_A, frictional force f_1, and reaction from the rod R_A. The forces on the B are m_Bg, normal reaction N_B on the lower surface, normal reaction N_A on the upper surface, tension T from the string, frictional force f_1 on the upper surface and frictional force f_2 on the lower surface. The forces on the block C are m_Cg, normal reaction N_C on the lower surface, normal reaction N_B on the upper surface, tension T from the string, frictional force f_2 on the upper surface and frictional force f_3 on the lower surface. Newton's second law on the block A, B, and C gives

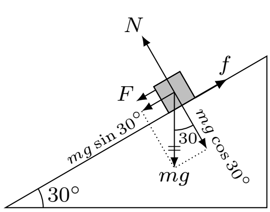
$$N_A = m_Ag, \tag{1}$$
$$N_B = N_A + m_Bg = (m_A + m_B)g, \quad \text{(using equation (1))} \tag{2}$$
$$N_C = N_B + m_Cg = (m_A + m_B + m_C)g. \quad \text{(using equation (2))} \tag{3}$$

The frictional forces in the sliding motion are equal to their limiting values i.e.,

$$f_1 = \mu N_A = \mu m_Ag, \quad \text{(using equation (1))} \tag{4}$$
$$f_2 = \mu N_B = \mu(m_A + m_B)g, \quad \text{(using equation (2))} \tag{5}$$
$$f_3 = \mu N_C = \mu(m_A + m_B + m_C)g. \quad \text{(using equation (3))} \tag{6}$$

Apply Netwton's second law on the block B and the block C to get the force F required to drag the block C at *constant speed*,

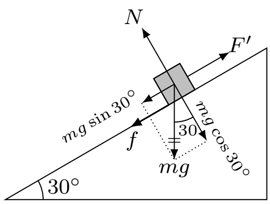
$$T = f_1 + f_2 = \mu m_Ag + \mu(m_A + m_B)g = 25\ \text{N}.$$
$$F = T + f_2 + f_3 = \mu(4m_A + 3m_B + m_C)g = 80\ \text{N}.$$



22. Let F be the force required to move the block downwards with a constant velocity. The forces on the block are its weight mg, normal reaction N, and frictional force f. The frictional force in the sliding motion is equal to its limiting value $f = f_{\max} = \mu N$. Resolve mg in the directions parallel and perpendicular to the incline. Apply Newton's second law to get

$$N = mg\cos 30°, \tag{1}$$
$$\begin{aligned} F &= f - mg\sin 30° = \mu N - mg\sin 30° && (\because f = \mu N) \\ &= mg(\mu\cos 30° - \sin 30°) && \text{(using equation (1))} \\ &= (2)(10)\left(\frac{\sqrt{3}}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 11.21\ \text{N}. \end{aligned}$$

Let F' be the force required to move the block upwards with a constant velocity. The direction of frictional force f is downwards and its magnitude is $f = f_{\max} = \mu N$. The normal reaction N is given by equation (1). Apply Newton's second law in the direction parallel to the incline to get



$$\begin{aligned} F' &= f + mg\sin 30° = \mu N + mg\sin 30° \\ &= mg(\mu\cos 30° + \sin 30°) \\ &= (2)(10)\left(\frac{\sqrt{3}}{\sqrt{2}}\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 31.21\ \text{N}. \end{aligned}$$