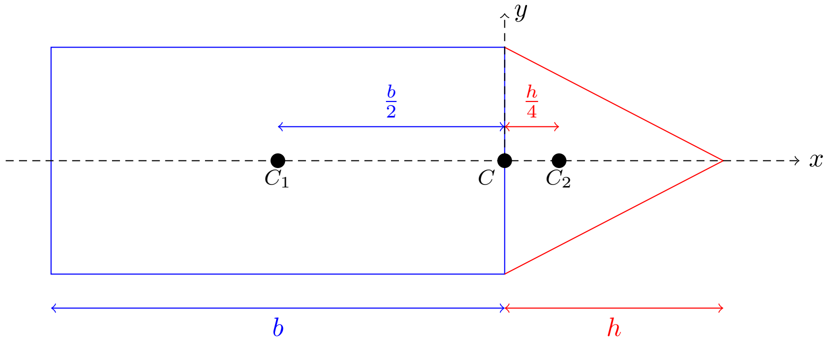
Question: A composite body is formed by joining a solid cylinder and a solid cone of the same radius. The length of the cylinder is b and height of the cone is h. If the centre of mass of the composite body is located in the plane between the solid cone and the solid cylinder then the ratio b/h is

(A) $1/\sqrt{6}$ (B) $1/2$ (C) $1/\sqrt{5}$ (D) $1/3$

Solution: Let the origin be located at the plane between the solid cone and the solid cylinder. Let x axis is along the symmetric axis towards the right.The centre of mass of the solid cylinder is at a distance $x_1 = -b/2$. The centre of mass of the solid cone is at a distance $x_2 = h/4$ (at distance h/4 from the base).



Let ρ be the mass density and r be the radius of the cylinder and the cone. The mass of the cylinder is $m_1 = \rho(\pi r^2 b)$ and mass of the cone is $m_2 = \rho(\frac{1}{3}\pi r^2 h)$. For the centre of mass of the composite body to be at origin,

$$x_{cm} = \frac{m_1 x1 + m_2 x_2}{m_1 + m_2} = \frac{\rho(\pi r^2 b)(-b/2) + \rho(\frac{1}{3}\pi r^2 h)(h/4)}{\rho(\pi r^2 b) + \rho(\frac{1}{3}\pi r^2 h)} = 0.$$

Simplify to get $b/h = 1/\sqrt{6}$.