Moseley's Law

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October 1, 2019

1 *Introduction*

The frequency ν of a characteristic X-ray of an element is related to its atomic number Z by

$$\sqrt{\nu} = a(Z - b),$$

where a and b are constants called proportionality and screening (or shielding) constants. For K series, the value of a is $\sqrt{3Rc/4}$ and that of b is 1. Here R is Rydberg's constant and c is speed of light (as in Bohr's model). For L series, the value of a is $\sqrt{5Rc/36}$ and b is 7.4. The relation and values of a and b are experimentally determined by Henry Moseley.

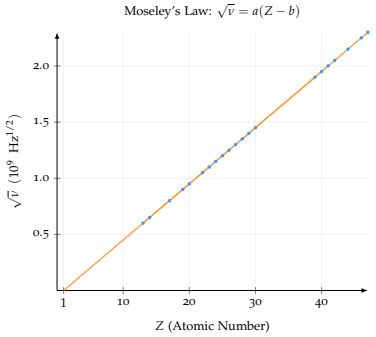

Figure 1: Relation between the frequency of characteristic x-ray and the atomic number Z. The line intersect the Z axis at $Z = b$ (b is 1 for K series and it is 7.4 for L series).

2 *Solved Problems on Moseley's Law*

Problem from IIT JEE 2003

Characteristic X-rays of frequency 4.2×10^{18} Hz are produced when transitions from L-shell to K-shell take place in a certain target material. Use Moseley's law to determine the atomic number of the target material. (Rydberg's constant $= 1.1 \times 10^7$ m^{-1}.)

Solution: The characteristic X-ray is emitted when an electron in L shell makes a transition to the vacant state in K shell. In Moseley's equation,

$$\sqrt{\nu} = a(Z - b),$$

the parameter $b \approx 1$ for this transition because electron from L shell finds nuclear charge Ze shielded by remaining one electron in K shell i.e., effective nuclear charge is $(Z-1)e$. Thus, by substituting values,

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{4.2 \times 10^{18}}{3 \times 10^8} = R(Z-1)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$
$$= 1.1 \times 10^7 (Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right],$$

which gives, $Z = 42$. Moseley's law played key role in arrangement of elements in the periodic table and to find many new (missing) elements.

Problem from IIT JEE 2014

If λ_{Cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_α X-ray line of molybdenum (atomic number 42), the the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to

(*A*) 1.99

(*B*) 2.14

(*C*) 0.50

(*D*) 0.48

Solution: The wavelength of K_α X-ray line is related to atomic number Z by Moseley's Formula

$$\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R(Z-1)^2.$$

Substitute the value of Z to get

$$\frac{\lambda_{Cu}}{\lambda_{Mo}} = \frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2} = \frac{(41)^2}{(28)^2} = 2.14.$$

The elements with higher atomic number (molybdenum in this example) gives high energy X-rays (short wavelengths).

Problem from IIT JEE 2008

Which of the following statements is wrong in the context of X-rays generated from a X-ray tube?

(*A*) Wavelength of characteristic X-rays decreases when the atomic number of the target increases.

(*B*) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.

(*C*) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube.

(*D*) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

Solution: The frequency ν of characteristic X-rays is related to atomic number Z by Moseley's law,

$$\sqrt{\nu} = a(Z-b),$$

which gives

$$\lambda = \frac{c}{\nu} = \frac{c}{a^2(Z-b)^2}.$$

Thus, the wavelength of emitted X-rays decreases with increase in Z. The cut-off wavelength of continuous X-rays corresponds to maximum energy of electron in X-ray tube. It is given by

$$hc/\lambda = eV,$$

where V is the accelerating potential. The intensity of X-rays depends on the number of electrons striking the target per second, which, in turn, depends on the electrical power given to the X-ray tube as energy of each electron is eV.

3 *Questions on Moseley's Law*

Question 1

If 178.5 pm is the wavelength of X-ray line of copper (atomic number 29) and 71 pm is the wavelength of the X-ray line of molybdenum (atomic number 42) then the value of a and b in Moseley's equation are

(*A*) $a = 7 \times 10^7\ \text{Hz}^{1/2}$, $b = 1.0$

(*B*) $a = 5 \times 10^7\ \text{Hz}^{1/2}$, $b = 7.4$

(*C*) $a = 5 \times 10^7\ \text{Hz}^{1/2}$, $b = 1.4$

(*D*) $a = 7 \times 10^7\ \text{Hz}^{1/2}$, $b = 7.4$

Answer: Substitute values of wavelength and atomic number in Moseley formula and solve.

Question 2

Moseley's Law for characteristic X-rays is $\sqrt{\nu} = a(Z - b)$. In this formula

(*A*) both a and b are independent on the material.

(*B*) a is independent but b depends on the material.

(*C*) b is independent but a depends on the material.

(*D*) both a and b depend on the material.

Answer: The constants a and b are independent of the material. They depends on X-ray series.

References

[1] Concepts of Physics Part 2 by HC Verma (Link to Amazon)

[2] IIT JEE Physics by Jitender Singh and Shraddhesh Chaturvedi

[3] X-ray Fluorescence and Moseley's Law (pdf article on X-rays, Moseley's Law and Moseley's Experiments)

[4] The Physical (in)significance of Moseley's Screening Parameters, (journal article in pdf by K Razi Naqvi)

[5] One hundred years of Moseley's law: An undergraduate experiment with relativistic effects (pdf)