On 'Amplitude' and 'RMS' Values

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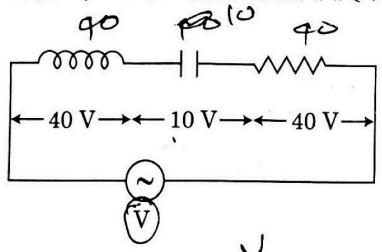
1 The Beginning

National Testing Agency (NTA) in India conducted NEET (UG) 2021. The following question is from Physics Paper (Code 02) Section A:

2. दिखाये गये चित्र के अनुसार एक L प्रेरकत्व का प्रेरक, एक C धारिता का धारित्र तथा एक 'R' प्रतिरोध का प्रतिरोधक 'V' वोल्ट विभवान्तर के प्रत्यावर्ती स्रोत से श्रेणी क्रम में जुड़े हैं।

L, C, R पर विभवान्तर क्रमशः 40 वोल्ट, 10 वोल्ट तथा 40 वोल्ट हैं। LCR श्रेणी परिपथ में प्रवाहित धारा $10\sqrt{2}$ एम्पियर है। परिपथ का प्रतिबाधा है :



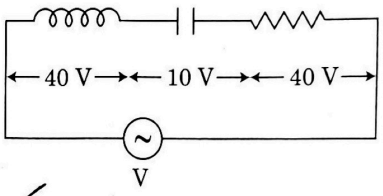
(1) 5 ओम

(2) $4\sqrt{2}$ ओम

(3) $5/\sqrt{2}$ ओम

(4) 4 ओम

2. An inductor of inductance L, a capacitor of capacitance C and a resistor of resistance 'R' are connected in series to an ac source of potential difference 'V' volts as shown in figure. Potential difference across L, C and R is 40 V, 10 V and 40 V, respectively. The amplitude of current flowing through LCR series circuit is $10\sqrt{2}$ A. The impedance of the circuit is :



(1) 5 Ω

(2) $4\sqrt{2}$ Ω

(3) $5/\sqrt{2}$ Ω

(4) 4 Ω

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The answer given by NTA is option (1) i.e., 5 Ω. This was challenged by students. In Hindi version of the question, the quantity 'amplitude of current' was translated as धारा instead of धारा का आयाम. However, NTA rejected the challenge. A writ petition was filed by a student Wajda Tabassum in the Supreme Court. A committee of three experts from IIT Guwahati, Delhi Technological University - Delhi and National Physical Laboratory - Delhi evaluated the solution. The committee concludes:

> "9. I submit that the Committee Members held a meeting on 27.11.2021 for the purpose of evaluation of solutions to Question No.2 of Section A in the Physics Paper of NEET (UG) 2021 both in Hindi and in English and concluded that the correct solution (answer) to said Question No.2 is 5Ω both in Hindi as well as English also the answer would be same i.e. 5Ω by providing the following explanation:
>
> (a) In Question, the word "धारा" should be understood as the amplitude of current which is also reflected from the circuit diagram as well as question statement, highlighting the alternating source.
>
> (b) In Question, "धारा" word is mentioned, which certainly does not mean the "rms" value. Any confusion can be easily clarified from the given circuit diagram."

A three judge bench of Supreme Court dismissed the petition on the basis of inputs from expert committee.

I am following IIT JEE and NEET due to my passion for solving physics problems. When Wajda Tabassuum approached me for comments, I could clearly see the injustice. However, injustice did not moved me as I was not affected. Mistakes happens, most of them gets corrected, a few remains, and the world moves on. What moved me was my own quest to know the following:

(i) Why NTA failed to see the mistake when it was pointed out? Unless mistakes are accepted, they cannot be corrected. Justifying it is a no solution.

(ii) Why simple matters are to be handled by the highest court? There are more complex and challenging legal cases for the Supreme Court.

(iii) Why committee of experts failed to do justice with their profession?

2 Refuting the Expert Committee

Let us see the statement 9(a) of the expert committee:

> "9 (a) In Question, the word धारा should be understood as the amplitude of current which is also reflected from the circuit diagram as well as question statement, highlighting the alternating source."

What I see from the question statement and the circuit diagram is:

(i) There is an alternating source V as mentioned in the question statement and shown in the circuit diagram (by symbol $-\!\!\sim\!\!-$)

(ii) It is a series LCR circuit with inductance L, capacitance C, and resistor R.

Hence, an alternating current will flow in the circuit. The current (धारा) $10\sqrt{2}$ एम्पियर can be interpreted as

(i) RMS value of alternating current (a convention followed worldwide including NCERT).

(ii) Amplitude of alternating current.

Now, coming to the statement 9(b) by the expert committee:

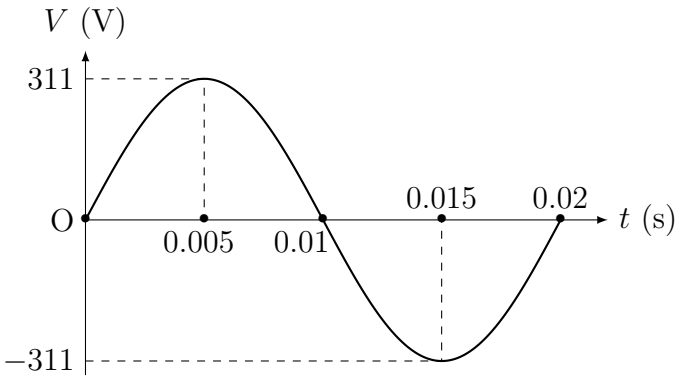
> "9 (b) In Question, धारा word is mentioned, which certainly does not mean the "rms" value. Any confusion can be easily clarified from the given circuit diagram."

This statement is insane. There is nothing in the question statement or circuit diagram to support this. The current (धारा) is an instantaneous quantity. It varies with time. Depending on the context, given value can be understood as its RMS value or Amplitude. The convention is in favour of RMS.

3 RMS and Amplitude

Let us go home (not to school or court) to understand the concept. All of us know about power supply in our home. It is AC. Its voltage is 220 V. Its frequency is 50 Hz. If we measure the voltage (actually potential difference) by connecting a voltmeter to a plug point, it displays 220 V. This is the RMS (Root Mean Square) value i.e., $V_{\text{rms}} = 220$ V.

Measuring or visualising frequency is not so easy. The voltage changes its polarity 50 times in a second i.e., one cycle in 0.02 s. The current changes its direction 50 times in a second. An oscilloscope is used to measure the frequency and display how voltage changes with time. If you connect an oscilloscope to a plug point in your home, its display will look like the following figure:



The voltage V varies with time t in a wave-like pattern. The voltage is zero at time $t = 0$, reaches a maximum value of 311 V at $t = 0.005$ s, then decreases to zero at $t = 0.01$ s, hereafter it changes sign, attains a value of -311 V at $t = 0.015$ s, and finally complete the cycle by attaining zero at $t = 0.02$ s. The waveform is sinusoidal. Its frequency is defined as the inverse of time taken to complete one cycle i.e.,

$$\nu = \frac{1}{0.02} = 50\,\text{Hz}.$$

The amplitude (or peak value or maximum value) of voltage is 311 V i.e., $V_0 = 311$ V. The amplitude V_0 and frequency ν are used to express voltage as a function of time as

$$V = V_0 \sin(2\pi\nu t) = 311 \sin(100\pi t).$$

Thus, voltage is completely defined by its frequency and amplitude. RMS is a useful mathematical construct given by

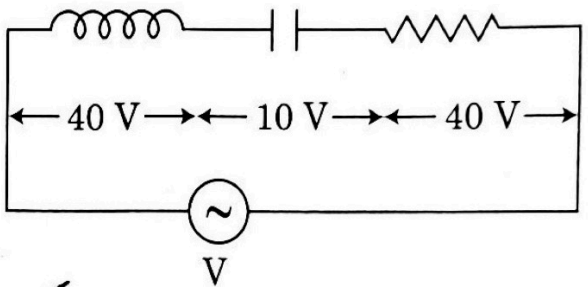
$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}.$$

RMS and amplitude are related by the above relation. One of them is good enough to specify the AC voltage. While specifying AC voltage, it is a good practice to mention whether specified value is the amplitude or the RMS value e.g., we should say 'the RMS value of voltage supply in our home is 200 V'. If not mentioned specifically, *convention* is to assume it as a RMS value.

Above concept is also true for alternating current.

4 Solution of the English Version

Question: An inductor of inductance L, a capacitor of capacitance C and a resistor of resistance R are connected in series to an ac source of potential difference V volts as shown in figure. Potential difference across L, C and R is 40 V, 10 V and 40 V, respectively. The amplitude of current flowing through LCR series circuit is $10\sqrt{2}$ A. The impedance of the circuit is:



Solution: The potential difference across L, C and R is 40 V, 10 V and 40 V, respectively. Neither question statement nor the circuit diagram explicitly mentioned whether given values are (i) RMS values or (ii) Amplitude. Hence, a student can interpret given values as

(i) RMS values (a convention followed worldwide including NCERT)

(ii) Amplitude

Given below is the solution for both cases.

Case (i):

RMS value of the potential differences across L, C and R is $V_{\mathrm{L,rms}} = 40$ V, $V_{\mathrm{C,rms}} = 10$ V, and $V_{\mathrm{R,rms}} = 40$ V. By using a result for series LCR circuit, we get RMS value of the source voltage as

$$\begin{aligned} V_{\mathrm{rms}} &= \sqrt{V_{\mathrm{R,rms}}^2 + (V_{\mathrm{L,rms}} - V_{\mathrm{C,rms}})^2} \\ &= \sqrt{40^2 + (40-10)^2} = \sqrt{2500} = 50 \text{ V}. \end{aligned}$$

The amplitude of current flowing through the LCR series circuit is $I_0 = 10\sqrt{2}$ A. Thus, RMS value of the current flowing through the circuit is

$$I_{\mathrm{rms}} = \frac{I_0}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10 \text{ A}.$$

The impedance of the given series LCR circuit is

$$Z = \frac{V_{\mathrm{rms}}}{I_{\mathrm{rms}}} = \frac{50}{10} = 5 \ \Omega.$$

NTA answer is the same. *It establishes, beyond any doubt, that NTA follows the convention of taking values as RMS (if not specifically mentioned).*

Case (ii):

Amplitude of the potential differences across L, C and R is $V_{\mathrm{L},0} = 40$ V, $V_{\mathrm{C},0} = 10$ V, and $V_{\mathrm{R},0} = 40$ V. Thus, Amplitude of the source voltage is

$$V_0 = \sqrt{V_{\mathrm{R},0}^2 + (V_{\mathrm{L},0} - V_{\mathrm{C},0})^2} = 50 \text{ V}.$$

The amplitude of current flowing through the LCR series circuit is $I_0 = 10\sqrt{2}$ A.

The impedance of the given series LCR circuit is

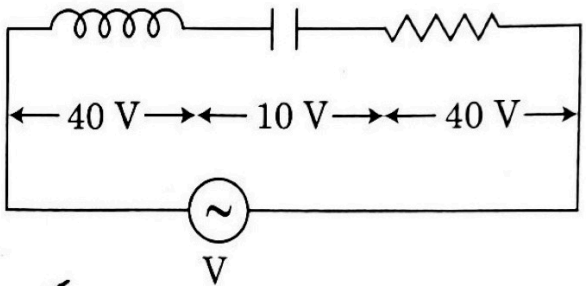
$$Z = \frac{V_{\mathrm{rms}}}{I_{\mathrm{rms}}} = \frac{V_0}{I_0} = \frac{50}{10\sqrt{2}} = \frac{5}{\sqrt{2}} \ \Omega.$$

This is one of the given option (3). *If convention of taking values as RMS* is not followed by NTA then option (3) should have been the correct answer. It re-establishes that NTA followed the convention.

5 Solution of the Hindi Version

Question: दिखाये गये चित्र के अनुसार एक L प्रेरकत्व का प्रेरक, एक C धारिता का धारित्र तथा एक R प्रतिरोध का प्रतिरोधक V वोल्ट विभवान्तर के प्रत्यावर्ती स्रोत से श्रेणी क्रम में जुड़े हैं।

L, C, R पर विभवान्तर क्रमशः 40 वोल्ट, 10 वोल्ट तथा 40 वोल्ट है। LCR श्रेणी परिपथ में प्रवाहित धारा $10\sqrt{2}$ एम्पियर है। परिपथ का प्रतिबाधा है:



Solution: Neither question statement nor the circuit diagram explicitly mentioned whether given potential differences (विभवान्तर) 40 वोल्ट, 10 वोल्ट तथा 40 वोल्ट or given current (धारा) of $10\sqrt{2}$ एम्पियर are (i) RMS values or (ii) Amplitude. Hence, a student can interpret given values as

(i) RMS values for both potential difference (विभवान्तर) and current (धारा) [a convention followed worldwide including NCERT and NTA ,as established in solution to the English version].

(ii) Amplitude for both potential difference (विभवान्तर) and current (धारा).

(iii) RMS value for potential difference (विभवान्तर) and Amplitude for current (धारा).

(iv) Amplitude for potential difference (विभवान्तर) and RMS value for current (धारा).

The students are expected to be consistent i.e., (s)he should take either RMS or Amplitude for both the potential difference (विभवान्तर) and the current (धारा). Interpretation (iii) and (iv) violates the expected consistency.

The solutions for all the four cases are given below:

Case (i):

RMS values for both potential difference (विभवान्तर) and current (धारा).

$$\begin{aligned} V_{\text{rms}} &= 50 \text{ V}, \\ I_{\text{rms}} &= 10\sqrt{2} \text{ A}, \\ Z &= \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{50}{10\sqrt{2}} = \frac{5}{\sqrt{2}} \ \Omega. \end{aligned}$$

This solution is in agreement with convention as well as expected consistency.

Case (ii):

Amplitude for both potential difference (विभवान्तर) and current (धारा).

$$\begin{aligned} V_0 &= 50 \text{ V}, \\ I_0 &= 10\sqrt{2} \text{ A}, \\ Z &= \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_0}{I_0} = \frac{50}{10\sqrt{2}} = \frac{5}{\sqrt{2}} \ \Omega. \end{aligned}$$

This solution is in agreement with consistency but not with convention.

Case (iii):

RMS value for potential difference (विभवान्तर) and Amplitude for current (धारा).

$$\begin{aligned} V_{\text{rms}} &= 50 \text{ V}, \\ I_0 &= 10\sqrt{2} \text{ A}, \\ Z &= \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_{\text{rms}}}{I_0/\sqrt{2}} = \frac{50}{10} = 5 \ \Omega. \end{aligned}$$

This solution is neither coherent with convention nor with consistency.

Case (iv):

Amplitude for potential difference (विभवान्तर) and RMS value for current (धारा).

$$\begin{aligned} V_0 &= 50 \text{ V}, \\ I_{\text{rms}} &= 10\sqrt{2} \text{ A}, \\ Z &= \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_0/\sqrt{2}}{I_{\text{rms}}} = \frac{50/\sqrt{2}}{10\sqrt{2}} = \frac{5}{2} \ \Omega. \end{aligned}$$

This solution is neither coherent with convention nor with consistency.

The solutions for four cases are sumarized in the following table.

Case	Convention	Consistency	Answer
(i)	✓	✓	$5/\sqrt{2}\ \Omega$
(ii)	×	✓	$5/\sqrt{2}\ \Omega$
(iii)	×	×	$5\ \Omega$
(iv)	×	×	$5/2\ \Omega$

To the best of my understanding, case (i) is the correct solution which is based on agreeable convention and consistency. NTA and expert committee goes with case (iii) which fails on both criteria.

6 Conclusion

AC voltage or current can be specified through RMS value or Amplitude. It is a good practice to mention whether specified value is the amplitude or the RMS value. If not mentioned specifically, *convention* is to assume it as a RMS value.

The English version of the question can have two answers (i) 5 Ω or (ii) $5/\sqrt{2}$ Ω, on the basis of interpretation of given potential differences as RMS values or Amplitudes. It is the RMS convention that leads to answer (i), which is reasonable, and I agree.

The Hindi version of the question have an answer $5/\sqrt{2}$ Ω, which is different from its English version. NTA or expert committee shall seriously look into this.