# *Specific Heats: $C_v$ and $C_p$ for Monatomic and Diatomic Gases*

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*October 21, 2019*

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## 1 Introduction

The molar specific heat capacity of a gas at constant volume $C_v$ is the amount of heat required to raise the temperature of 1 mol of the gas by 1°C at the constant volume. Its value for monatomic ideal gas is $3R/2$ and the value for diatomic ideal gas is $5R/2$.

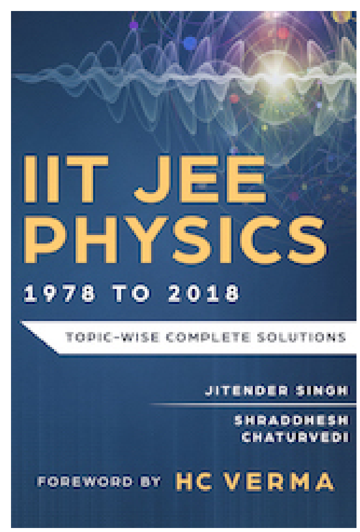
The molar specific heat of a gas at constant pressure ($C_p$ is the amount of heat required to raise the temperature of 1 mol of the gas by 1°C at the constant pressure. Its value for monatomic ideal gas is $5R/2$ and the value for diatomic ideal gas is $7R/2$.

| | Monatomic | Diatomic |
|---|---|---|
| $f$ | 3 | 5 |
| $C_v$ | $3R/2$ | $5R/2$ |
| $C_p$ | $5R/2$ | $7R/2$ |
| $\gamma$ | 1.66 | 1.4 |

The specific heat at constant volume is related to the internal energy $U$ of the ideal gas by

$$C_v = \left.\frac{\mathrm{d}U}{\mathrm{d}T}\right|_v = \frac{f}{2}R,$$

where $f$ is degrees of freedom of the gas molecule. The degrees of freedom is 3 for monatomic gas and 5 for diatomic gas (3 translational + 2 rotational). The internal energy of an ideal gas at absolute temperature $T$ is given by $U = fRT/2$.




The specific heat at constant pressure ($C_p$) is greater than that at constant volume ($C_v$). The heat given at constant volume is equal to the increase in internal energy of the gas. The heat given at constant pressure is equal to the increases in internal energy of the gas plus the work done by the gas due to increase in its volume ($Q = \Delta U + \Delta W$). The difference between $C_p$ and $C_v$ is given by Mayer's formula

$$C_p - C_v = R.$$

The ratio of the specific heats, also called adiabatic index, is given by

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}.$$

The ratio of the specific heats is 5/3 for monatomic ideal gas and 7/5 for diatomic gas. Its value for air is 1.4. This ratio is used to define (1) adiabatic process $pV^\gamma =$ const, and (2) speed of sound in gases $v = \sqrt{\gamma RT/M}$.

## 2 *Solved Problems on Specific Heats of Gases*

### *Problem from IIT JEE 2012*

Two moles of ideal helium gas are in rubber balloon at 30 °C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35 °C. The amount of heat required in raising the temperature is nearly, (Take $R = 8.31$ J/mol-K).

(*A*) 62 J

(*B*) 104 J

(*C*) 124 J

(*D*) 208 J

**Solution:** The pressure inside the balloon is equal to the constant ambient pressure because balloon is fully expandable. Thus, given process is isobaric, so heat exchanged in the process is given by,

$$\Delta Q = nC_p\Delta T.$$

The helium is a monoatomic gas with three degrees of freedom ($f = 3$). Thus, the specific heat at constant volume ($C_v$) and constant pressure ($C_p$) for helium are given by,

$$C_v = (f/2)R = (3/2)R,$$
$$C_p = C_v + R = (5/2)R.$$

Substitute the values to get,

$$\Delta Q = (2)(5/2)(8.31)(35 - 30) \approx 208 \text{ J}.$$

### *Problem from IIT JEE 2009*

$C_v$ and $C_p$ denotes the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then,

(*A*) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas.

(*B*) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas.

(*C*) $C_p/C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas.

(*D*) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas.

**Solution:** The molar heat capacities of an ideal gas having $f$ degrees of freedom are given by,

$$C_v = R(f/2), \quad (1)$$
$$C_p = C_v + R = R(f+2)/2. \quad (2)$$

Use above equations to get,

$$C_p + C_v = (1+f)R,$$
$$\frac{C_p}{C_v} = 1 + \frac{2}{f},$$
$$C_p \cdot C_v = \frac{f(2+f)}{4}R^2.$$

The value of $f$ for monoatomic and diatomic gases are 3 and 5. Thus, B and D are correct.

*Problem from IIT JEE 1998*

Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is

(*A*) 30 K

(*B*) 18 K

(*C*) 50 K

(*D*) 42 K

**Solution:** The piston in the cylinder A is free to move. Hence pressure of the gas is constant and the heat is given to it at constant pressure i.e.,

$$Q_A = nC_p\Delta T_A.$$

The piston of the cylinder B is fixed. Hence the volume of the gas is constant and the heat is given at constant volume i.e.,

$$Q_B = nC_v\Delta T_B.$$

The ratio of specific heats for a diatomic gas is $C_p/C_v = 7/5$. The heat given to the two gases is equal, $Q_A = Q_B$. Substitute values to get

$$\Delta T_B = \left(C_p/C_v\right)\Delta T_A = 42 \text{ K}.$$

*Problem from IIT JEE 1990*

When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is

(A) 2/5

(B) 3/5

(C) 3/7

(D) 5/7

**Solution:** The change in the internal energy of an ideal gas, when its temperature changes by $\Delta T$, is given by

$$\Delta U = nC_v \Delta T.$$

The heat supplied at constant pressure, to increase the gas temperature by $\Delta T$, is given by

$$\Delta Q = nC_p \Delta T.$$

Divide the first equation by the second to get

$$\frac{\Delta U}{\Delta Q} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{5}{7},$$

where $\gamma = 7/5$ is the ratio of specific heats for a diatomic gas.

*Problem from IIT JEE 1988*

If one mole of a monatomic gas ($\gamma = 5/3$) is mixed with one mole of a diatomic gas ($\gamma = 7/5$), the value of $\gamma$ for the mixture is

(A) 1.4

(B) 1.5

(C) 1.53

(D) 3.07

**Solution:** The internal energy of one mole of an ideal gas at temperature $T$ is

$$U = \frac{f}{2}RT = C_v T,$$

where $f$ is the degrees of freedom and $C_v$ is the molar specific heat at constant volume. The degrees of freedom for a monatomic gas is three and that for a diatomic gas is five. Thus, the internal energy of a monatomic gas is $U_m = 3RT/2$ and that of a diatomic gas is

$U_d = 5RT/2$. The mixture contains the two moles of gases with the total internal energy

$$\begin{aligned} U_{\text{mix}} &= U_m + U_d \\ &= \tfrac{3}{2}RT + \tfrac{5}{2}RT = 4RT. \end{aligned}$$

Thus, the internal energy of *one* mole of the mixture is $U_{\text{mix}}/2 = 2RT$. Hence, the molar specific heat at constant volume is $C_{v,\text{mix}} = 2R$. The molar specific heat at constant pressure is $C_{p,\text{mix}} = C_v + R = 3R$. The ratio of specific heats for the mixture is $\gamma_{\text{mix}} = C_{p,\text{mix}}/C_{v,\text{mix}} = 3/2$. </p> <p> We encourage you to derive the expression for $\gamma_{\text{mix}}$ when $n_1$ moles of an ideal gas with $f_1$ degrees of freedom are mixed with $n_2$ moles of another ideal gas with $f_2$ degrees of freedom:

$$\gamma_{\text{mix}} = \frac{n_1(f_1+2) + n_2(f_2+2)}{n_1 f_1 + n_2 f_2}.$$

## 3 *Questions on Specific Heats of Gases*

### *Question 1*

70 cal of heat is required to raise the temperature of 2 mole of an ideal diatomic gas at constant pressure from 30 degree Celsius to 35 degree Celsius. The amount of heat required to raise the temperature of the same gas through the same range (30 degree Celsius to 35 degree Celsius) at constant volume is

(*A*) 30 cal

(*B*) 50 cal

(*C*) 70 cal

(*D*) 90 cal

**Answer:** (B)

### *Question 2*

At a given temperature, the specific heat of a gas at a constant pressure is always greater than its specific heat at constant volume. (True/False)

(*A*) True

(*B*) False

**Answer:** (A) See Mayer's Formula.

## *References*

[1] IIT JEE Physics by Jitender Singh and Shraddhesh Chaturvedi

[2] Concepts of Physics Part 2 by HC Verma (Link to Amazon)

[3] Specific Heats of Gases, School Physics

[4] Specific Heats of Gases, Theory and Experiments. Download pdf.

[5] Specific Heat, MIT